

# Efficiency of Nano-Gypsum and Elemental Sulfur Application in Reclaiming Saline-Alkali Soils in Siwa Oasis, Egypt

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## ABSTRACT

The calcareous, salt-affected soils of Egypt's Siwa Oasis, characterized by shallow groundwater and marginal irrigation, create combined risks of sodicity and salinity, severely limiting crop recovery. This study aimed to identify the optimal, cost-effective application rates of nano-gypsum (NG) and elemental sulfur ( $S^0$ ) for achieving six-month reclamation targets. We evaluated cost per unit ESP reduction and per 0.01 increase in  $SF^*$ , accounting for dose-response and timing differences. Field trials compared NG (120, 240, 480 kg ha<sup>-1</sup>) and  $S^0$  (0.5, 1.0, 2.0 t ha<sup>-1</sup>) on non-saline sodic (NSS) and saline-sodic (SS) soils across sandy, loam, and clay-loam textures. The amendments were incorporated to 0–20 cm, followed by immediate leaching. Soil plots were sampled at 1, 3, and 6 months to measure chemical (pH, E<sub>Ce</sub>, ESP, SAR) and physical (hydraulic conductivity, water retention, available water) properties. The structural factor (SF) and its normalized form ( $SF^*$ ) were subsequently derived. Statistical analysis was performed using a three-way factorial RCBD with repeated measures (Texture × Treatment × Time) and ANOVA with Fisher's LSD ( $\alpha = 0.05$ ). In NSS soils, all textures successfully reached an ESP < 15% within six months, accompanied by coherent declines in SAR/E<sub>Ce</sub> and gains in hydraulic conductivity (ks) and structural factor, confirming functional desodification. In SS soils, both amendments reduced ESP, SAR, and E<sub>Ce</sub>; however, progress was texture-limited. While sandy and loam soils achieved ESP ≈ 14–15%, the clay-loam soils remained above the sodicity threshold (>15%) at six months, with E<sub>Ce</sub> levels persistently above 4 dS m<sup>-1</sup>, indicating a need for continued leaching. Economic analysis revealed that NG rates of 240–480 kg ha<sup>-1</sup> were consistently on the cost-effectiveness frontier for both  $\Delta ESP$  and  $\Delta SF^*$ . Conversely,  $S^0$  at 0.5–1.0 t ha<sup>-1</sup> was dominated, and the 2.0 t ha<sup>-1</sup> rate was only occasionally non-dominated, with steep incremental cost-effectiveness ratios (ICERs, i.e., higher costs per unit improvement). We recommend prioritizing NG at 240–480 kg ha<sup>-1</sup> to secure ESP reduction and structural recovery within six months.  $S^0 \leq 2$  t ha<sup>-1</sup> should be reserved for carbonate-rich, fine-textured soils to sustain acid dissolution. These applications must be paired with immediate leaching, maintained drainage, and extended leaching cycles in SS soils until the target E<sub>Ce</sub> < 4 dS m<sup>-1</sup> is met.

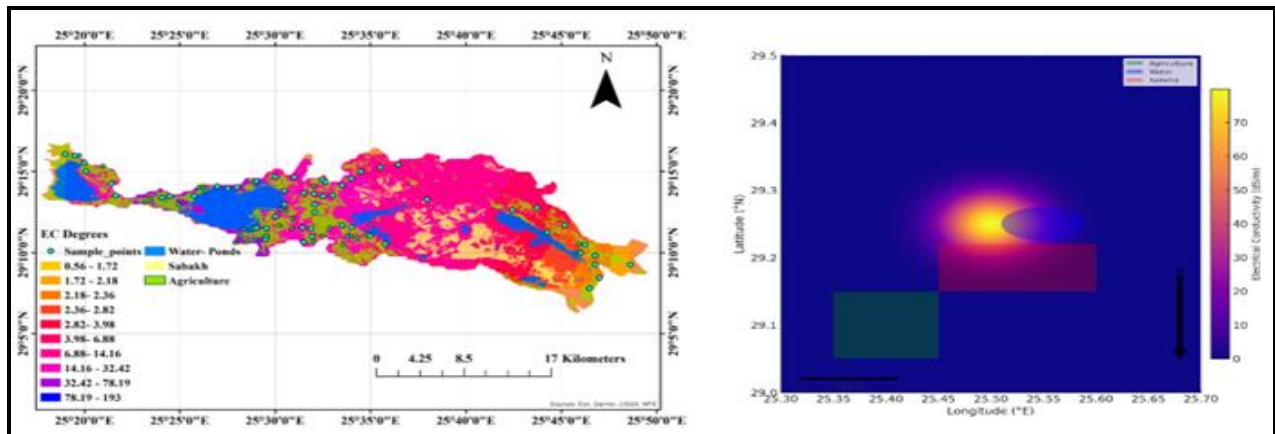
**Key words:** Siwa Oasis; saline-sodic soils; non-saline sodic soils; nano-gypsum; elemental sulfur.

## INTRODUCTION

Soil salinity and sodicity are among the most serious constraints to sustainable agriculture worldwide, particularly in arid and semi-arid regions where irrigation is essential. It is estimated that 20–30% of irrigated lands are affected by salt accumulation, leading to reduced yields, soil degradation, and in severe cases, land abandonment (Rengasamy, 2010 and Qadir *et al.*, 2014). In such regions, high evaporative demand, shallow saline groundwater, and poor leaching exacerbate salt buildup, while inadequate soil and water management intensify risks. Scientifically, saline soils are defined by electrical conductivity (E<sub>Ce</sub>) > 4 dS m<sup>-1</sup>, while sodic soils have an exchangeable sodium percentage (ESP) ≥ 15% (Richards, 1954; Abrol *et al.*, 1988 and El-Ramady *et al.*, 2024). Sodicity disperses clay colloids, destabilizes aggregates, reduces infiltration, and limits hydraulic conductivity. When sodicity co-occurs with salinity, osmotic stress further restricts plant water and nutrient uptake (Shainberg & Letey, 1984 and Munns & Tester, 2008). Egypt's Western Desert, particularly the Siwa Oasis, exemplifies this dual challenge. A shallow water table, limited drainage, and prolonged irrigation have driven secondary salinization and sodification. Remote-sensing assessments show that saline soils expanded from ~35 km<sup>2</sup> to ~64 km<sup>2</sup>, and waterlogged areas from ~19 km<sup>2</sup> to ~51 km<sup>2</sup> between 1992 and 2015 (Fig. 1) (Elnaggar *et al.*, 2017). This expansion threatens both traditional farming and ecological balance, highlighting the urgency of reclamation strategies tailored to Siwa's calcareous soils and hydrogeologic constraints. Calcium-based amendments are the cornerstone of sodic-soil reclamation because Ca<sup>2+</sup> replaces exchangeable Na<sup>+</sup>, promoting flocculation and aggregate stability (El-Mowelhi *et al.*, 1976). The displaced sodium is leached downward as sodium sulfate under proper irrigation and drainage (Wang *et al.*, 2021). In Egypt, gypsum (CaSO<sub>4</sub>·2H<sub>2</sub>O) is widely used, but its limited solubility under alkaline conditions and high application requirements often slow reclamation and increase costs (Shainberg *et al.*, 1989).

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**Fig. 1. Distribution of non-saline and saline soils based on electrical conductivity threshold ( $EC_e > 4 \text{ dS m}^{-1}$ ) in Siwa Oasis between 1992 and 2015**

Nano-gypsum (NG), with its higher surface area and reactivity, dissolves more readily than conventional gypsum, providing more available  $\text{Ca}^{2+}$  for exchange. Controlled studies report that NG applications as low as  $240 \text{ kg ha}^{-1}$  can reduce ESP by  $> 90\%$ , lower soil pH by  $\sim 1$  unit, and double hydraulic conductivity compared with conventional gypsum (Kumar & Thiyageshwari, 2018; Salama *et al.*, 2022; Abd El-Halim *et al.*, 2023 and El-Henawy *et al.*, 2024). Elemental sulfur ( $\text{S}^0$ ), in contrast, acts indirectly: upon microbial oxidation, it generates  $\text{H}_2\text{SO}_4$ , dissolving native  $\text{CaCO}_3$  and releasing  $\text{Ca}^{2+}$ . This process reduces ESP, neutralizes alkalinity, and improves structure, but proceeds gradually and depends on temperature and moisture (Dahnke, 1988; Tabatabai, 2005 and Wang *et al.*, 2021).

Thus, NG offers rapid improvement, while  $\text{S}^0$  supports slower, long-term recovery. Although gypsum and  $\text{S}^0$  are widely used in Egypt (El-Sheref *et al.*, 2019; Amer *et al.*, 2023 and Ali *et al.*, 2024), NG remains largely experimental, reported mostly from pot and greenhouse studies (Salama *et al.*, 2022; Abd El-Halim *et al.*, 2023 and El-Henawy *et al.*, 2024). To date, direct, field-scale comparisons between NG and  $\text{S}^0$  under Siwa's specific conditions (calcareous soils, shallow groundwater, poor drainage, and variable textures) are lacking, despite clear evidence of salinity and sodicity expansion (Elnazer *et al.*, 2022 and Salem & Jia, 2024).

This study addresses this gap through multi-texture, field-scale evaluation of NG and  $\text{S}^0$  over six months. The objectives were to:

1. quantify the effects of nano-gypsum and elemental sulfur on some soil chemical (pH,  $EC_e$ , ESP, SAR) and physical ( $K_s$ ,  $\theta_{-33}$ ,  $\theta_{-1500}$ , AW) properties, and on selected structure indices (AI, SF,  $\text{SF}^*$ ).
2. compare between amendments' efficacy in relation to soil textures and sampling times.

3. identify optimal application rates that achieve reclamation targets ( $\text{ESP} < 15\%$ , improved  $K_s$ , AW, and  $\text{SF}^*$ ) maximize cost-effectiveness.
4. find out a practical, effective and economic recommendation for Siwa-like conditions, considering soil amendment, leaching requirements and water-quality constraints.

## MATERIAL AND METHODS

### 1. Area Study

The field experiment was conducted in Siwa Oasis, a closed depression in Egypt's Western Desert ( $29^\circ 00' - 29^\circ 30' \text{ N}$ ,  $25^\circ 15' - 26^\circ 08' \text{ E}$ ). The oasis extends  $\sim 80 \text{ km}$  east-west, covers  $\sim 1,000 - 1,100 \text{ km}^2$ , and lies mainly below sea level ( $-17$  to  $-20 \text{ m}$  in the central basin, reaching  $-23 \text{ m}$  in local minima) (Fig. 2). The climate is hyper-arid, with mean annual rainfall of  $\sim 10 - 20 \text{ mm}$ , very hot summers ( $\geq 38^\circ \text{ C}$  daytime), and cool winters (night minima  $\sim 5 - 7^\circ \text{ C}$ ). Surface soils are sandy to loamy sand, highly calcareous ( $\text{CaCO}_3$  ranging from a few percent to several tens of percent), and are mapped mainly as Typic Torripsamments and Haplosalids, including gypsic variants (Elnaggar *et al.*, 2017). Agriculture depends on springs and groundwater, but intensive irrigation, shallow water tables, and limited drainage have caused secondary salinization, sodification, and waterlogging (Abdel Rahman *et al.*, 2019).

### 2. Experimental Design

The study compared nano-gypsum (NG) and elemental sulfur ( $\text{S}^0$ ) for reclaiming alkaline salt-affected soils across textures. Based on USS/FAO criteria, two soil categories were selected: non-saline sodic (NSS:  $EC_e < 4 \text{ dS m}^{-1}$ ,  $\text{ESP} \geq 15\%$ ) and saline-sodic (SS:  $EC_e \geq 4 \text{ dS m}^{-1}$ ,  $\text{ESP} \geq 15\%$ ). Within each category, a factorial randomized complete block design

(RCBD) with three replicates was established, combining Texture (3 levels: sandy, loam, clay loam) and Treatment (7 levels: control; NG at 120, 240, 480 kg ha<sup>-1</sup>; S<sup>0</sup> at 500, 1000, 2000 kg ha<sup>-1</sup>). This yielded 63 plots per category (3 textures × 7 treatments × 3 replicates), totaling 126 plots. Plots measured 4 × 5 m (20 m<sup>2</sup>), separated by bunds (~30 cm), buffer strips (1 m), and alleys (2 m) to minimize lateral flow. Amendments were broadcast and incorporated into the 0–20 cm layer, followed by an initial irrigation (~300 mm; ≈6 m<sup>3</sup> per plot) to dissolve amendments and initiate leaching, consistent with FAO/USSL guidelines. Irrigation water quality (EC, SAR) was monitored during the experiment. Soil and core samples were collected at 1, 3, and 6 months for chemical (pH, ECe, ESP, SAR) and physical (bulk density, Ks, water retention, AW) analyses. Randomization was computer-generated, and plots were oriented across slope.

### 3. Analytical procedures

For each plot and sampling time (1, 3, and 6 months), composite soil samples from the 0–20 cm layer (five subsamples per fixed grid) were collected and homogenized. A portion was air-dried, sieved (<2 mm), and used for chemical analysis, while undisturbed cores were preserved for physical measurements. All analyses were run in duplicate; samples exceeding a relative percent difference of 5% (chemistry) or 10% (physical) were reanalyzed. Chemical analyses: Saturated soil pastes were prepared with CO<sub>2</sub>-free deionized water and equilibrated for at least 4 h. Extracts were analyzed following standard procedures. Electrical conductivity (ECe) was measured using a conductivity meter (Hanna Instruments HI98331 Soil Test™), calibrated daily and automatically temperature-corrected to 25 °C (Rhoades, 1996). Soil pH was determined in the paste extract using a glass-electrode pH meter (Hanna Instruments HI 99121), standardized daily at pH 4.00, 7.00, and 10.00 (Rhoades, 1996). Soluble cations (Na<sup>+</sup>, K<sup>+</sup>, Ca<sup>2+</sup>, Mg<sup>2+</sup>) were quantified by ion chromatography according to ASTM D6919 (ASTM, 2009), with ICP-OES (brand & model) used as confirmatory analysis. Soluble anions (SO<sub>4</sub><sup>2-</sup>, HCO<sub>3</sub><sup>-</sup>, CO<sub>3</sub><sup>2-</sup>) were determined by suppressed ion chromatography using a Thermo Scientific Dionex ICS-5000 system, equipped with an anion suppressor, following ASTM D4327 (ASTM, 2011). Quality control included method blanks, calibration checks, and certified standards; ionic charge balance was required to close within ±5%, otherwise samples were re-run (Soil Survey Staff, 2014). Exchangeable Na<sup>+</sup> was extracted with 1 M NH<sub>4</sub>OAc (pH 7.0) and quantified by ion chromatography, while cation-exchange capacity (CEC) was determined using NH<sub>4</sub>OAc saturation/displacement steps, expressed in cmolc kg<sup>-1</sup> (Sumner and Miller, 1996). ESP was calculated as:

$$\text{ESP} = \frac{\text{Exchangeable Na}^+ \times 100}{\text{CEC}}$$

Sodium adsorption ratio (SAR) was calculated from saturation extracts as:

$$\text{SAR} = \frac{[\text{Na}^+]}{\sqrt{[\text{Ca}^{++}] + [\text{Mg}^{++}]}}/2$$

with concentrations expressed in mmolc L<sup>-1</sup> (Abrol *et al.*, 1988 and Soil Survey Staff, 2014). Bulk density (Bd) was measured on undisturbed cores (≈5 cm diameter) as oven-dry mass (105 °C, 24 h) divided by core volume (Blake and Hartge, 1986). Saturated hydraulic conductivity (Ks) was measured on undisturbed cores using a constant-head permeameter under low hydraulic gradient (0.05–0.1) to ensure laminar flow, with falling-head checks in fine textures to confirm Darcy's law (Klute and Dirksen, 1986). Ks values were corrected to a reference temperature of 20–25 °C using viscosity ratios. Soil water retention was determined on 100 cm<sup>3</sup> undisturbed cores at –33 kPa (field capacity, θ<sub>-33</sub>) and –1500 kPa (permanent wilting point, θ<sub>-1500</sub>) using a pressure-plate extractor (Soil Moisture 1500F2, PV15) following the procedures of Klute (1986). Available water (AW) was calculated as the difference (θ<sub>-33</sub> – θ<sub>-1500</sub>). Treatment means were computed as block averages (n = 3) within each texture. Sampling time was analyzed as a repeated factor in a linear mixed model (appropriate covariance structure selected per fit diagnostics). Post-hoc comparisons used Fisher's LSD at α = 0.05 (Gomez & Gomez, 1984 and Littell *et al.*, 2006).

### 4. Indices and normalization [Structure Factor (SF); Normalized Structure Factor (SF\*); and Aggregation Index (AI)]

To synthesize treatment effects on soil physical condition while avoiding cross-texture bias, we computed three unitless indices following composite-indicator best practices (z-standardization, polarity handling, and linear rescaling) as recommended by the OECD–JRC Handbook (2008).

#### Structure Factor (SF)

SF was defined from measured variables (Ks, AW, ESP) as a study-specific composite.

SF = (K<sub>s</sub> × AW) / (1 + ESP/100), with K<sub>s</sub> in cm h<sup>-1</sup>; AW = θ<sub>-33</sub>–θ<sub>-1500</sub> in % v/v.

#### Normalized Structure Factor (SF\*)

SF\* normalized SF to the concurrent control within each texture and time (control = 1.00).

$$\text{SF}^*_{\text{treat, texture, t}} = \text{SF}_{\text{control, texture, t}} / \text{SF}_{\text{treat, texture, t}}$$

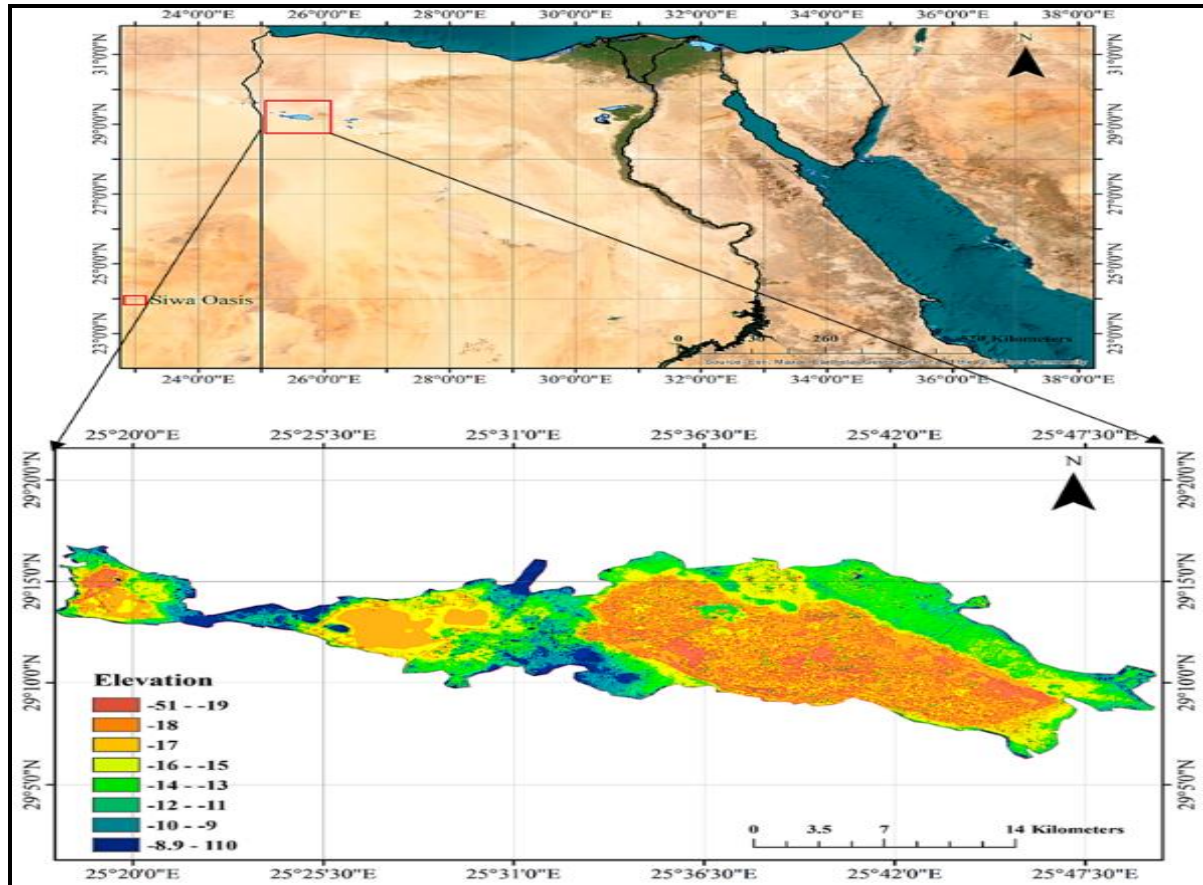


Fig. 2. Location map of Siwa Oasis in Egypt's Western Desert (29°00'–29°50' N and 25°20'–25°47' E)

### Aggregation Index (AI).

AI aggregates standardized Ks and AW positively and ESP negatively, then linearly rescaled to 35–100 with the control fixed at 35 in each texture × time set. This standardized–aggregated–rescaling workflow mirrors established soil-quality frameworks such as SMAF and the broader use of derived physical indices (e.g., Dexter's S-index) in soil physics. The negative contribution of ESP reflects its well-documented role in degrading soil structure and hydraulic conductivity in sodic conditions (Andrews *et al.*, 2004 and Dexter, 2004).

#### Step 1 (standardize):

$$K_s' = K_s \cdot \bar{K}_s / s_{K_s}, \quad AW' = AW \cdot \bar{AW} / s_{AW}, \quad ESP' = ESP \cdot \bar{ESP} / s_{ESP}$$

(bars  $\bar{\phantom{x}}$  and  $s$  are the mean and SD within that texture × time across treatments, including control).

#### Step 2 (composite score):

$$S = K_s' + AW' - ESP'/3$$

Where S expresses positive contributions from Ks and AW and a negative contribution from ESP. The 1/3 down-weights ESP value so it doesn't dominate).

#### Step 3 (rescale to 35–100; control = 35).

Within each texture × time, S is linearly transformed such that the control is 35.00 and the best S is 100

$$AI = 35 + (S - S_{control}) / (S_{max} - S_{control}) \times (100 - 35)$$

If  $S_{max} = S_{control}$ , AI is set to 35 for all treatments at that texture × time. AI is reported to two decimals.

### 5. Irrigation water sampling and analysis

Irrigation water was sampled at each soil-sampling interval (1, 3, and 6 months). At each interval. Three grab samples (≈500 mL each) were collected at the field inlet in pre-rinsed HDPE bottles, stored at 4 °C, and analyzed within 48 h. Samples were filtered and analyzed for electrical conductivity (EC) and soluble cations. EC was measured at 25 °C with a calibrated conductivity meter, and reported in dS m<sup>-1</sup>. Sodium adsorption ratio (SAR) was computed from Na<sup>+</sup>, Ca<sup>2+</sup>, and Mg<sup>2+</sup> concentrations (mmol L<sup>-1</sup>) determined by ICP-OES (Standard Methods 3120 / APHA, 2005). Interval-wise summaries (mean ± SD, n) were reported.

#### 1. Economic assessment (cost–effectiveness)

The study evaluated private (farm-level) variable costs over the experimental period, aligning the

effectiveness endpoint with the 6-month soil assessment used throughout the study. For each option—nano-gypsum, NG (120, 240, 480 kg ha<sup>-1</sup>) and elemental sulfur S (0.5, 1.0, 2.0 t ha<sup>-1</sup>)—the total variable cost per hectare was:

$$\text{Cost}_{\text{treatment}} = \text{Material} + \text{Transport} + \text{Application (EGP ha}^{-1}\text{)}$$

with “Application” defined as machinery + labor for one pass (additional passes at higher rates).

Co-primary endpoints were: (i)  $\Delta\text{ESP}$  (decrease vs. concurrent control at 6 months) and (ii)  $\Delta\text{SF}^*$  (increase vs. concurrent control at 6 months).

$$\Delta\text{ESP} = \text{ESP}_{\text{control}} - \text{ESP}_{\text{treat}} \text{ (positive=improvement), } \Delta\text{SF}^* = \text{SF}^*_{\text{treat}} - \text{SF}^*_{\text{control}}$$

ESP is interpreted against standard sodicity thresholds (Richards, 1954 and Abrol *et al.*, 1988).

**Incremental analysis and dominance rules:** Options were ordered by effect and evaluated sequentially to compute incremental cost ( $\Delta\text{Cost}$ ), incremental effect ( $\Delta\text{Eff}$ ) and ICER ( $\Delta\text{Cost}/\Delta\text{Eff}$ ). Treatments that were strictly dominated (higher cost and no greater effect) were excluded. After removing strictly dominated options, it assessed extended (weak) dominance by checking that ICERs along the effectiveness-ordered set were non-decreasing; options violating monotonicity were ruled out. The remaining non-dominated set defined the cost–effectiveness frontier (NICE, 2025).

#### ICER formulas (evaluated at 6 months):

$$\text{ICER}_{\text{ESP}} = \text{Cost}_{\text{treat}} - \text{Cost}_{\text{control}} / \Delta\text{ESP} \text{ (EGP per 1 ESP-point reduced),}$$

$$\text{ICER}_{\text{SF}^*} = \text{Cost}_{\text{treat}} - \text{Cost}_{\text{control}} / \Delta\text{SF}^* / 0.01 \text{ (EGP per 0.01 SF}^*\text{)}. \text{}$$

These follow the standard definition of an incremental cost-effectiveness ratio (difference in costs divided by difference in outcomes).

**Sensitivity analysis:** We performed one-way  $\pm 25\%$  sensitivity analyses on material price, application cost per pass, transport, and other variable items and examined the stability of frontier membership and ICER magnitudes (Drummond *et al.*, 2015).

**Optional value-of-effect (cost–benefit):** When a willingness-to-pay  $\lambda$  (EGP ha<sup>-1</sup> per ESP-point) is specified for a target crop and price scenario, a net-benefit form can be calculated:

$$\text{Net Benefit} = \lambda \times \Delta\text{ESP} - \Delta\text{Cost}$$

**Valuation parameter ( $\lambda$ ):** For sensitivity and net-benefit analyses, we inform the willingness-to-pay per unit ESP reduction ( $\lambda$ , EGP ha<sup>-1</sup> per ESP-point) using the threshold–slope (broken-line) yield model. The Maas–Hoffman formulation is the standard for crop response to salinity (ECe), providing a crop-specific threshold and slope (% yield loss per unit above the threshold). The same piecewise-linear approach has

been explicitly applied to sodicity (ESP), with published threshold ESP and slope (% yield change per ESP unit) for multiple crops. Accordingly,  $\lambda$  can be approximated as:  $\lambda = s / 100 \times (Y \times P)$ , where  $s$  is the crop’s %-yield change per 1 ESP unit (from the ESP threshold–slope model),  $Y$  is expected yield (t ha<sup>-1</sup>), and  $P$  is the farm-gate price (EGP t<sup>-1</sup>). This valuation is secondary to the primary CEA and is reported only in sensitivity analyses (Maas & Hoffman, 1977 and Gupta & Sharma, 1990).

#### Statistical analysis (concise)

Soil responses were analyzed separately for non-saline sodic (NSS) and saline-sodic (SS) soils using a three-way factorial RCBD with replicates: fixed factors were Texture (sandy, loam, clay-loam), Treatment (control; nano-gypsum 120/240/480 kg ha<sup>-1</sup>; sulfur 0.5/1.0/2.0 t ha<sup>-1</sup>), and Time (1, 3, 6 months) with their interactions; Blocks was randomized and nested within texture, and the plot served as the repeated subject (split-plot-in-time). Repeated-measures covariance was modeled as compound symmetry with Huynh–Feldt adjustment, or AR (1) when favored by AIC. Assumptions were checked on studentized residuals (Q–Q for normality, Levene’s within Texture  $\times$  Time for homogeneity, Cook’s  $D$  for influence); monotone transforms were used as needed (e.g., log Ks,  $\sqrt{\text{ECe}}$ ). Significance was set at  $\alpha = 0.05$ ; significant omnibus effects were followed by Fisher’s LSD (0.05). For consistency, one LSD per variable per soil category (from the pooled residual MS) was applied to Texture  $\times$  Time treatment means; when interactions were significant, simple effects (Treatment within a Texture  $\times$  Time) used the same error term and  $\alpha$ . Analyses were performed in CoHort v6.400 (or equivalent mixed/repeated-measures software) (Snedecor and Cochran, 1989).

## RESULTS AND DISCUSSION

### A. Irrigation-Water Quality Over Time

Measured EC values of irrigation water increased from  $3.20 \pm 0.64$  to  $4.25 \pm 0.62$  dS m<sup>-1</sup> between months 1 and 6 (averaged 3.75 dS m<sup>-1</sup>,  $n = 9$ ), and SAR increased from  $5.50 \pm 0.65$  to  $7.08 \pm 0.55$  (averaged 6.26,  $n = 9$ ). Replicate variability was modest: EC, the coefficient of variation (CVs) was 20%, 13%, and 15% at 1, 3, and 6 months, respectively; SAR CVs were 12%, 10%, and 8%, respectively (Table 1). According to the FAO water quality guidelines (Ayers and Westcot, 1985), measured ECw  $\approx 3\text{--}4$  dS m<sup>-1</sup> indicates a moderate–high salinity hazard (manage with leaching and tolerant crops), while SAR  $\approx 5\text{--}7$  reflects a low–moderate sodium hazard.

**Table 1. Electrical conductivity (EC) and sodium adsorption ratio (SAR) of irrigation-water at different sampling intervals (mean  $\pm$  SD).**

Interval (months)	n	EC (dS m <sup>-1</sup> ), mean $\pm$ SD	EC range (dS m <sup>-1</sup> )	SAR mean $\pm$ SD	SAR range
1	3	3.20 $\pm$ 0.64	2.27–3.90	5.50 $\pm$ 0.65	4.33–6.20
3	3	3.80 $\pm$ 0.50	3.20–4.45	6.20 $\pm$ 0.60	5.40–6.95
6	3	4.25 $\pm$ 0.62	3.50–5.46	7.08 $\pm$ 0.55	6.30–8.08
Overall	9	3.75	2.27–5.46	6.26	4.33–8.08

Note: Same irrigation supply applied to all plots across textures and soil categories; values are inlet-water quality, therefore not stratified by texture.

The combined EC–SAR context suggests limited infiltration risk during application (higher EC promotes flocculation), but sustained use can still increase soil ESP without chemical amendment or leaching management (Ayers and Westcot, 1985). Thus, upward trends in EC and SAR would tend to work against improvements in ESP and structure, reinforcing both the necessity and demonstrated efficacy of gypsum/sulfur.

### **B. Integrated chemical and physical responses of non-saline sodic soils to nano-gypsum and sulfur**

#### **Soil Chemical Properties**

Soil pH decreased significantly with both nano-gypsum (NG) and sulfur (S) across all textures, with the greatest effects observed at higher amendment rates and longer incubation periods (Table 2). In sandy soil (8% CaCO<sub>3</sub>), the highest applications (NG-480 and S-2000) lowered pH from 8.58 to 8.12 by month 6 (~5% acidification). This was accompanied by marked reductions in soluble HCO<sub>3</sub><sup>-</sup> and CO<sub>3</sub><sup>2-</sup>, consistent with the microbial oxidation of S to H<sub>2</sub>SO<sub>4</sub> (S<sup>0</sup> + 1.5 O<sub>2</sub> + H<sub>2</sub>O → H<sub>2</sub>SO<sub>4</sub>) and subsequent CaCO<sub>3</sub> dissolution, which released Ca<sup>2+</sup> into solution; concomitant increases in SO<sub>4</sub><sup>2-</sup> further confirmed this pathway (Tabatabai, 2005 and Mahdy *et al.*, 2017). Amendments also reduced sodicity indicators. Exchangeable Na<sup>+</sup> declined by ~30% under NG-480 and ~28% under S-2000, ESP by ~30% (20.4 → 14.2%), SAR by ~26% (14.2 → 10.6), and ECe by 27–30% compared with the control. These changes demonstrate efficient Na<sup>+</sup>–Ca<sup>2+</sup> exchange and leaching, driven by the immediate supply of soluble Ca<sup>2+</sup> from NG and the sustained Ca<sup>2+</sup> release from S-induced acidification. Significant amendment  $\times$  texture  $\times$  period interactions (Table 2; Figs. 3–4) confirmed that amelioration accelerated with time (1→6 months) and was most rapid in sandy soils due to their lower CEC and higher leaching efficiency. These outcomes align with recent studies reporting that nano-gypsum at low doses (120–960 kg ha<sup>-1</sup>) reduces sodicity more effectively than bulk gypsum, owing to its high reactivity and faster Ca<sup>2+</sup> release (Patle *et al.*, 2022 and Salama *et al.*, 2022). In calcareous soils, elemental sulfur lowers pH, dissolves carbonates, and increases Ca<sup>2+</sup> activity, while stimulating microbial activity that

enhances sulfur oxidation and Na<sup>+</sup> displacement (Malik *et al.*, 2021 and Al-Mayahi *et al.*, 2024). The efficiency of this process is strongly influenced by CaCO<sub>3</sub> content. In this study, loam (15% CaCO<sub>3</sub>) and clay loam (28% CaCO<sub>3</sub>) showed slower pH reduction than sandy soil (8% CaCO<sub>3</sub>), reflecting the greater buffering capacity of finer-textured, carbonate-rich soils. This agrees with reports that high CaCO<sub>3</sub> levels delay sulfur-driven acidification and require higher rates or longer incubation to achieve significant sodicity reduction (Tabatabai, 2005 and Elgala *et al.*, 2021). Comparable ESP reductions of 20–40% within 3–6 months have been widely documented with gypsum applied at 2–5 t ha<sup>-1</sup> (Zhao *et al.*, 2018 and Bello *et al.*, 2021). Long-term studies also confirm that gypsum decreases SAR and ESP while improving infiltration and structural stability, especially when combined with sulfur in calcareous soils (Green *et al.*, 2023; Singh *et al.*, 2023 and Xiao *et al.*, 2025). Taken together, NG provides a rapid Ca<sup>2+</sup> pulse to initiate Na<sup>+</sup> exchange and double-layer collapse, while S maintains acidification and continuous Ca<sup>2+</sup> release through ongoing CaCO<sub>3</sub> dissolution. This combined effect explains the observed sequence—declines in pH, exchangeable Na<sup>+</sup>, ESP, SAR, and ECe—which establishes the chemical foundation for subsequent physical recovery (Ks, AI, SF\*). The stronger early response in sandy soils compared with clay loam reflects differences in buffering capacity and water movement, underscoring the need for higher or repeated amendment doses in fine-textured soils (Rezapour *et al.*, 2023).

#### **Soil Physical Properties**

The improvements in soil chemistry (declines in pH, ESP, SAR, and carbonate alkalinity, accompanied by increased SO<sub>4</sub><sup>2-</sup>) were closely reflected in soil physical responses across all three textures (Table 3; Figs. 5–6). Bulk density remained statistically unchanged ( $\approx$ 1.30–1.58 g cm<sup>-3</sup>), confirming that amelioration resulted from pore reorganization and aggregate stabilization rather than compaction relief. K<sub>d</sub> increased significantly ( $p \leq 0.05$ ) with both amendment rate and incubation time, confirming the role of Ca<sup>2+</sup> in restoring soil permeability. In sandy soil (8% CaCO<sub>3</sub>), K<sub>d</sub> increased from 3.56 cm h<sup>-1</sup> in the control to 8.40 cm h<sup>-1</sup> at 6

months under NG-480 and S-2000, a 2.4-fold increase. Loam soils (15%  $\text{CaCO}_3$ ) showed a doubling of  $K_s$  ( $1.02 \rightarrow 2.40 \text{ cm h}^{-1}$ ), while clay loam (28%  $\text{CaCO}_3$ ) rose from 0.25 to  $0.60 \text{ cm h}^{-1}$ . Though absolute values were smallest in clay loam, the relative gain ( $\sim 2.4\times$ ) was comparable across textures, showing that even highly buffered, fine-textured soils can respond when  $\text{Ca}^{2+}$  supply is sufficient. These improvements are agronomically critical because they restore infiltration, enhance leaching, and improve root-zone aeration. The faster increase in sandy soils reflects rapid  $\text{Na}^+$  replacement and intrinsic permeability, while clay loam required longer reaction times for  $\text{Ca}^{2+}$  to displace  $\text{Na}^+$  and reestablish pore connectivity, leaving it near the sodicity threshold after six months. AW ( $\theta_{-33} - \theta_{-1500}$ ) increased modestly but significantly ( $\approx 0.3\text{--}0.6$  percentage points,  $\sim 8\%$  relative gain). For example, in loam soil AW increased from 11.67% to 12.67% by month 6. These small but significant ( $p \leq 0.05$ ) changes indicate that improved aggregation enhanced field

capacity and mesopore continuity. Even small increases in AW are valuable in arid zones, as they improve irrigation efficiency and plant-available water (Yu *et al.*, 2022 and Xiao *et al.*, 2025). The strongest improvements were observed in structural indicators. In sandy soil, the Aggregation Index (AI) increased from 35% (control) to 84–100% and  $\text{SF}^*$  from  $\sim 1.0$  to  $\sim 3.0$  after 6 months under NG-480 and S-2000. Loam and clay loam also improved significantly ( $p \leq 0.05$ ), with AI increasing from 35 to 85–100 and  $\text{SF}^*$  roughly doubling ( $1.0 \rightarrow 2.0\text{--}2.8$ ) by month 6. These results mark the transition from sodicity-dispersed, unstable structures to water-stable aggregates capable of sustaining infiltration, aeration, and root penetration (Green *et al.*, 2023; Singh *et al.*, 2023 and Xiao *et al.*, 2025). The slower response in loam and clay loam reflects stronger  $\text{CaCO}_3$  buffering and higher clay content, which delay  $\text{Na}^+$  displacement and aggregate stabilization.

**Table 2. ANOVA significance and mean values of soil chemical properties in non-saline sodic soils amended with nano-gypsum and sulfur**

F	pH	ECe (dS $\text{m}^{-1}$ )	CEC (cmolc $\text{kg}^{-1}$ )	Exch. Na (cmolc $\text{kg}^{-1}$ )	ESP (%)	SAR	$\text{HCO}_3^-$	$\text{CO}_3^{2-}$ meq/L	$\text{SO}_4^{2-}$
Amendments (A)	***	***	NS	***	***	***	**	**	**
– Control	8.68	3.34	16.0	3.38	20.40	14.23	3.46	0.29	12.99
– NG-120	8.57	2.94	16.0	3.07	18.41	13.04	3.29	0.27	12.35
– NG-240	8.50	2.71	16.0	2.76	16.56	11.94	3.08	0.26	11.54
– NG-480	8.43	2.45	16.0	2.37	14.23	10.55	2.81	0.23	10.53
– S-500	8.54	2.83	16.0	2.90	17.47	12.48	3.16	0.26	11.84
– S-1000	8.48	2.57	16.0	2.59	15.60	11.36	2.91	0.24	10.90
– S-2000	8.43	2.34	16.0	2.44	14.69	10.82	2.83	0.24	10.62
LSD <sub>0.05</sub> (A)	0.03	0.10	0.40	0.07	0.60	0.60	0.20	0.05	0.90
Texture (T)	***	**	NS	***	***	***	**	**	**
– Sandy	8.40	2.44	10.0	1.43	14.26	10.55	2.61	0.22	9.78
– Loam	8.50	2.68	16.0	2.54	15.84	11.51	2.99	0.25	11.21
– Clay loam	8.60	2.92	22.0	4.18	19.02	13.41	3.51	0.29	13.18
LSD <sub>0.05</sub> (T)	0.02	0.08	0.30	0.06	0.50	0.50	0.15	0.04	0.70
Period (P)	***	**	NS	***	***	***	**	**	**
– 1 month	8.64	3.20	16.0	3.23	19.47	13.68	3.46	0.29	12.97
– 3 months	8.51	2.72	16.0	2.75	16.61	11.97	3.12	0.26	11.69
– 6 months	8.33	2.04	16.0	2.08	12.54	9.52	2.47	0.20	9.24
LSD <sub>0.05</sub> (P)	0.02	0.07	0.30	0.05	0.40	0.40	0.12	0.03	0.60
Interactions									
A $\times$ T	**	*	NS	**	**	**	*	*	*
A $\times$ P	**	*	NS	**	**	**	*	*	*
T $\times$ P	**	*	NS	**	**	**	*	*	*
A $\times$ T $\times$ P	*	*	NS	*	**	*	*	*	*

Note: The table summarizes ANOVA results, including main effects (amendments, textures, and periods) and their interactions for soil chemical. Mean values are presented for seven treatments, three soil textures, and three incubation periods. LSD<sub>0.05</sub> is reported separately for amendments (A), textures (T), and periods (P). NS = non-significant; \* =  $P \leq 0.05$ ; \*\* =  $P \leq 0.01$ ; \*\*\* =  $P \leq 0.001$ .

### Integrated significance.

The consistent, statistically significant improvements in  $K_s$ , AW, AI, and SF\* confirm that NG and S amendments restore both soil chemistry and physical function. Once ESP dropped below ~15%, soils shifted from impermeable, dispersive conditions to stable, well-aggregated states. The agronomic significance lies in enhanced infiltration, salt leaching,

water storage, and aeration—functions essential for sustainable crop growth. From a management standpoint, sandy soils can be reclaimed rapidly with single applications, loams require moderate effort, and clay loams demand higher or repeated inputs with extended leaching. This texture-dependent pattern underscores the importance of site-specific reclamation strategies (Rezapour *et al.*, 2023 and Singh *et al.*, 2023).

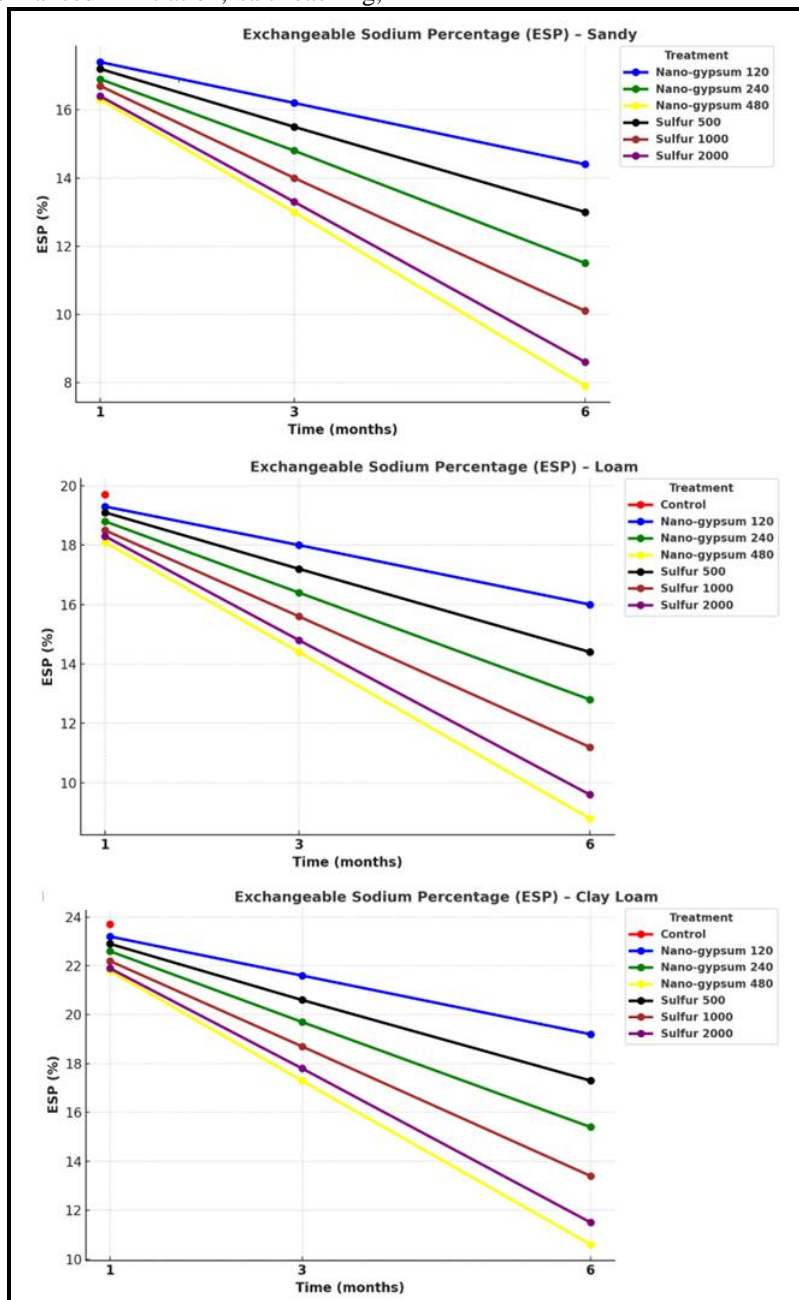


Fig. 3. Effect of nano-gypsum and elemental sulfur amendments on soil exchangeable sodium percentage (ESP, %) at three-time intervals (1, 3, and 6 months after application) in non-saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

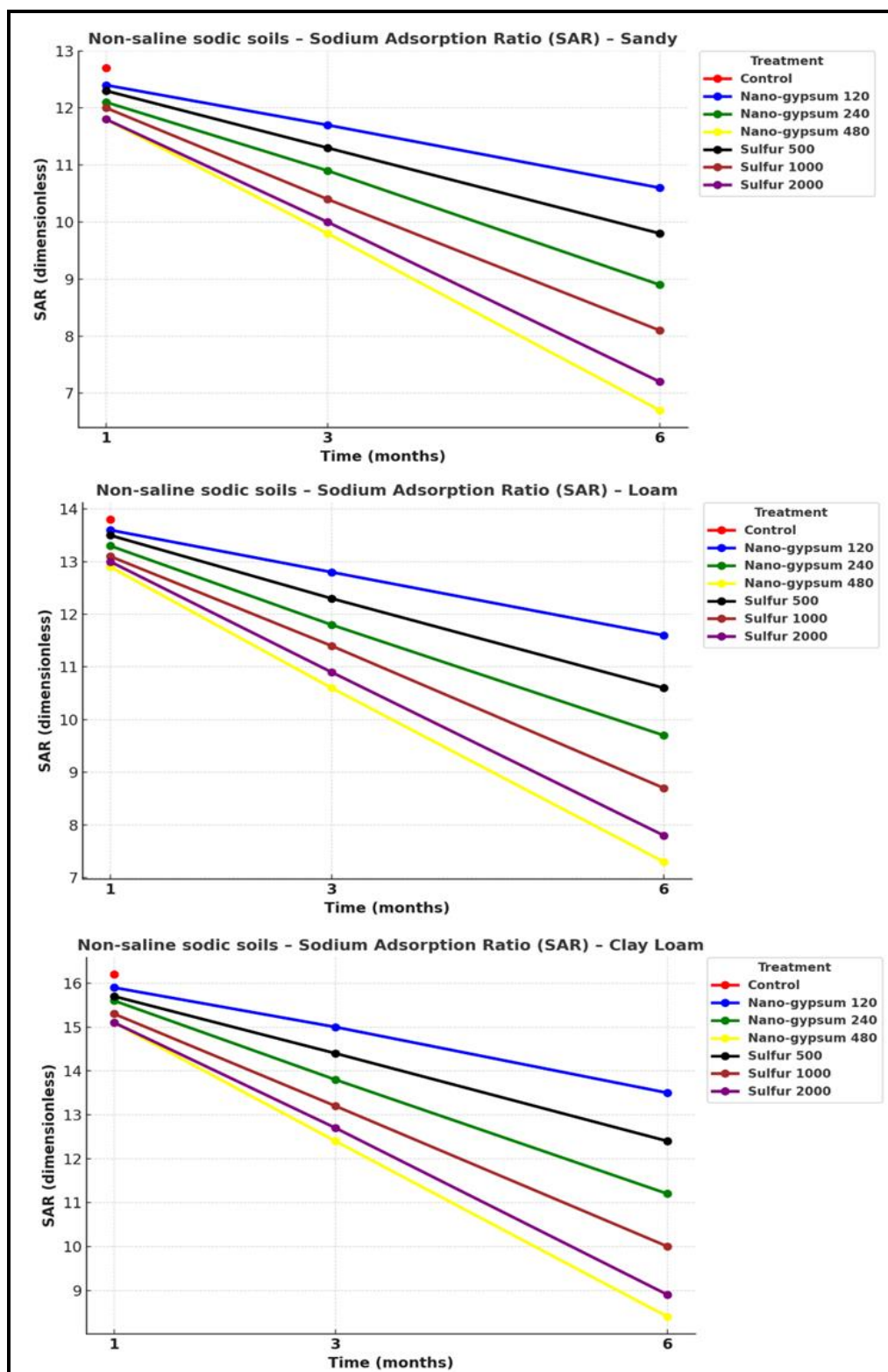


Fig. 4. Effect of nano-gypsum and elemental sulfur amendments on Sodium adsorption ratio (SAR) at three-time intervals (1, 3, and 6 months after application) in non-saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

**Table 3. ANOVA significance and mean values of soil physical properties in non-saline sodic soils amended with nano-gypsum and sulfur**

Variation	Bd (g m <sup>-3</sup> )	Ks (cm h <sup>-1</sup> )	$\theta_{-33}$ (%)	$\theta_{-1500}$ (%)	AW (%)	AI	SF	SF*
Amendments (A)	***	***	**	**	***	***	**	**
– Control	1.43	1.61	21.03	9.37	11.67	35.00	11.75	1.00
– NG-120	1.43	2.03	21.35	9.38	11.98	47.29	15.41	1.30
– NG-240	1.44	2.40	21.35	9.38	11.98	65.14	18.60	1.57
– NG-480	1.44	2.95	21.35	9.38	11.98	90.38	23.57	1.99
– S-500	1.43	2.11	21.76	9.42	12.33	67.28	17.18	1.45
– S-1000	1.43	2.46	21.76	9.42	12.33	85.96	20.47	1.73
– S-2000	1.43	2.81	21.76	9.42	12.33	99.40	23.68	2.00
LSD <sub>0.05</sub> (A)	0.04	0.25	0.30	0.30	0.30	3.00	0.80	0.10
Texture (T)	***	***	***	***	***	***	***	***
– Sandy	1.58	5.22	11.07	3.53	7.54	73.13	35.12	1.66
– Loam	1.42	1.49	23.40	9.86	13.54	73.08	17.70	1.60
– Clay loam	1.55	0.37	30.40	14.86	15.54	73.63	4.95	1.63
LSD <sub>0.05</sub> (T)	0.04	0.25	0.30	0.30	0.30	3.00	0.80	0.10
Period (P)	***	***	***	**	***	***	***	***
– 1 month	1.43	1.80	21.20	9.37	11.84	77.63	13.50	1.15
– 3 months	1.43	2.32	21.53	9.37	12.17	70.85	18.52	1.57
– 6 months	1.43	3.06	22.20	9.53	12.67	70.63	26.71	2.25
LSD <sub>0.05</sub> (P)	0.04	0.25	0.30	0.30	0.30	3.00	0.80	0.10
Interactions								
A × T	**	**	*	*	**	**	*	*
A × P	**	**	*	*	**	**	*	*
T × P	**	**	*	*	**	**	*	*
A × T × P	*	*	NS	NS	*	*	*	*

**Note:** The table summarizes ANOVA results, including main effects (amendments, textures, and periods) and their interactions for soil physical properties. Mean values are presented for seven treatments, three soil textures, and three incubation periods. LSD<sub>0.05</sub> values are reported separately for amendments (A), textures (T), and periods (P). NS = non-significant; \* =  $P \leq 0.05$ ; \*\* =  $P \leq 0.01$ ; \*\*\* =  $P \leq 0.001$ .

### C. Six-Month Cost–Benefit and Cost-Effectiveness of NG vs S<sup>0</sup> in Non-Saline Sodic Soils

We assessed sodicity relief over six months using a willingness-to-pay parameter ( $\lambda$ , EGP·ha<sup>-1</sup> per 1-point ESP reduction) and ranked treatments by Net Monetary Benefit (NMB =  $\lambda \times \Delta\text{ESP} - \text{Cost}$ ). This framework avoids ratio pathologies and enables consistent ranking at fixed  $\lambda$ , applying standard dominance and extended-dominance rules to remove options that are more costly and less effective, or whose ICERs exceed those of superior alternatives. Conceptually,  $\lambda$  represents the revenue at risk from sodicity, linked to crop response, price, and area.

Following Maas & Hoffman (1977); Rengasamy (2010) and Qadir *et al.* (2014), we used  $\lambda \approx 200$  (tolerant/low-margin systems),  $\lambda \approx 500$  (typical cereals/mixed systems), and  $\lambda \geq 1000$  EGP·ha<sup>-1</sup> per ESP-point (high-value/sensitive crops or severe infiltration constraints). Across these scenarios, nano-gypsum (NG) consistently produced positive NMB in

sandy, loam, and clay loam soils, even at conservative  $\lambda$ . By contrast, elemental sulfur (S<sup>0</sup>) was negative or near break-even within six months due to higher application costs and the slower oxidation-driven release of Ca<sup>2+</sup>. For example, in sandy soils, NMB for NG-1/NG-2/NG-3 reached  $\approx +1,340/+2,430/+3,510$  EGP·ha<sup>-1</sup> at  $\lambda = 500$  and  $\approx +3,040/+5,580/+8,460$  EGP·ha<sup>-1</sup> at  $\lambda = 1000$ , while S-1/S-2/S-3 remained negative or marginal even at higher  $\lambda$ ; similar patterns were observed in loam and clay loam (Tables 4–5).

Cost-effectiveness analysis (Table 6, Fig. 7) showed that NG-2 and NG-3 were always the most efficient options across all soil textures, while S-1 and S-2 were less effective and more costly (dominated). S-3 sometimes appeared competitive because it reduced ESP more (non-dominated), but only at a steep ICER, its high cost made it uneconomical in the short term. A clear dose/equivalence contrast underpins these results: NG was applied at 120–480 kg·ha<sup>-1</sup> (up to 960 kg·ha<sup>-1</sup> in comparable trials), which is 1–2 orders of magnitude lower than typical conventional gypsum applications

( $\text{t}\cdot\text{ha}^{-1}$ ). Where  $\text{S}^0$  substitutes for gypsum, a stoichiometric requirement of  $\text{SR} \approx 0.19 \times \text{GR}$  (mass basis) is reasonable; for sulfuric acid,  $\sim 0.61 \text{ t H}_2\text{SO}_4$  is roughly equivalent to 1 t gypsum (excluding dilution water).

Overall, applying the NMB framework to our six-month dataset shows that NG-1/NG-2/NG-3 form the cost-effectiveness frontier at  $\lambda = 200\text{--}500$ , with S-1 and S-2 dominated and S-3 only occasionally non-

dominated in some textures due to larger effect size but only at a steep ICER within six months. Hence, *\*moderate-high NG rates (240–480  $\text{kg}\cdot\text{ha}^{-1}$ ) are the most efficient six-month strategy for lowering ESP and improving soil structure ( $F^*$ ,  $K^*$ )*. Elemental sulfur becomes attractive mainly over longer periods and/or under very high  $\lambda$ , justified by sensitive crops and high revenues at risk.

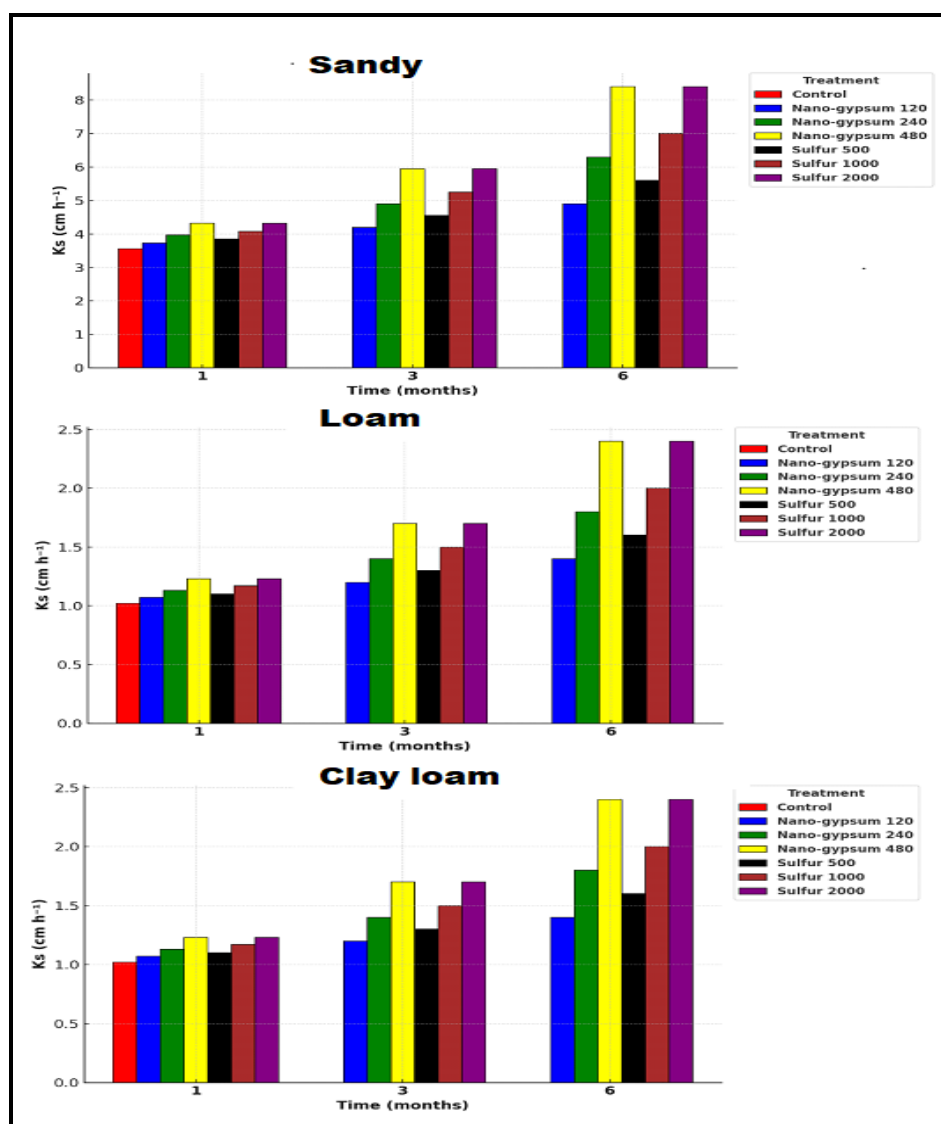


Fig. 5. Effect of nano-gypsum and elemental sulfur amendments on Saturated hydraulic conductivity ( $K_s$ ,  $\text{cm h}^{-1}$ ) at three-time intervals (1, 3, and 6 months after application) in non-saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

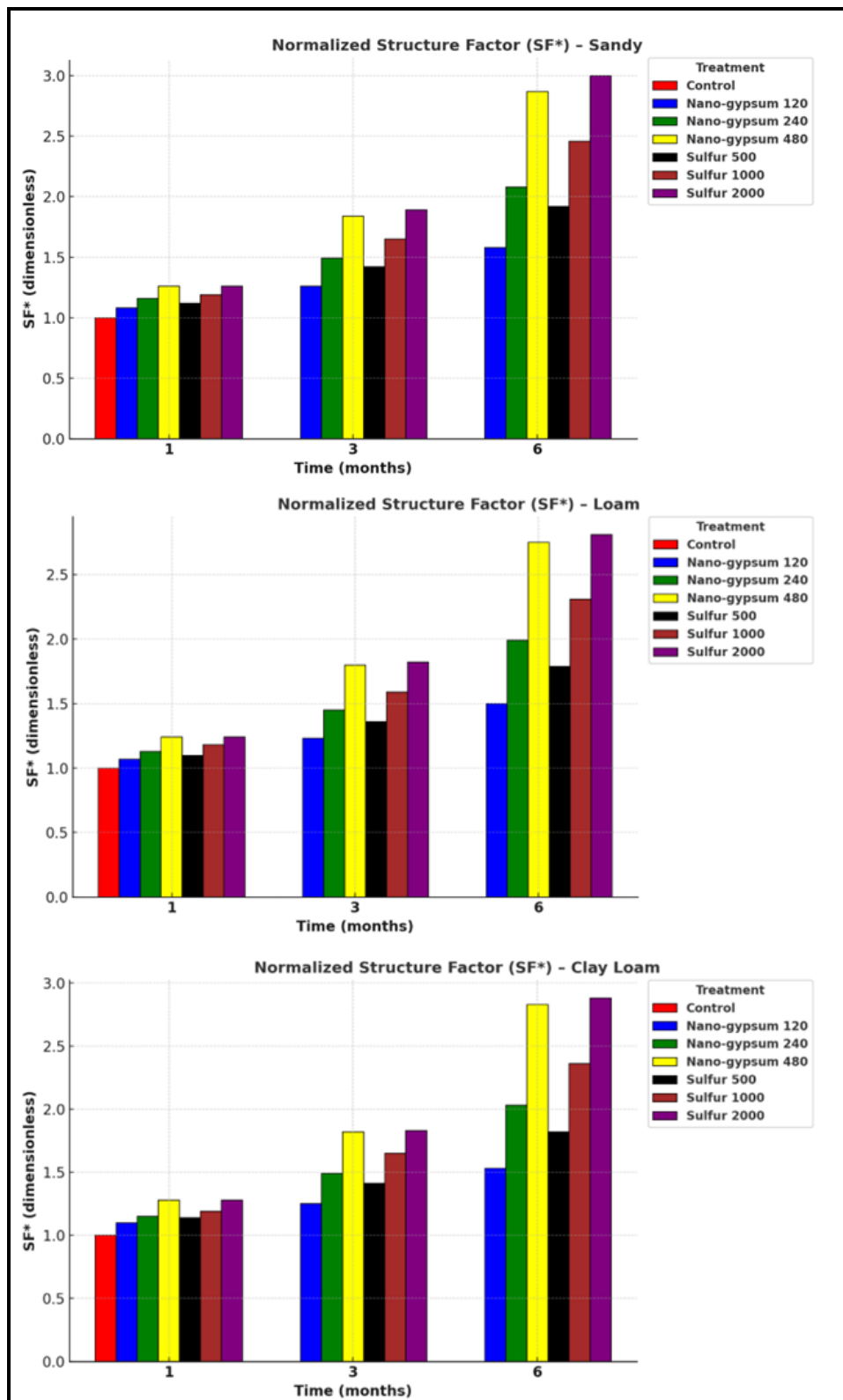


Fig. 6. Effect of nano-gypsum and elemental sulfur amendments on normalized structure factor (SF\*) at three-time intervals (1, 3, and 6 months after application) in non-saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

**Table 4. Cost–benefit of nano-gypsum and sulfur applications ( $\lambda = 200$ ) in non-saline sodic soils (sandy, loam, clay loam) after 6 months**

Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate $\times$ price) (EGP/ha)	Estimated ESP, %	$\Delta$ ESP, % (ESP <sub>control</sub> – ESP <sub>treatment</sub> )	Cost per 1 % ESP decreased (EGP) (Total cost / $\Delta$ ESP)	Benefit (EGP/ha) at $\lambda=200$ (200 $\times$ $\Delta$ ESP)	Net Benefit (EGP/ha) at $\lambda=200$ (Benefit – Cost)
Sandy Soil								
CK (Control)	-	-	-	17.80	-	-	-	-
NG-1	0.12		360	14.40	3.40	105.88	680.00	320.00
NG-2	0.24	3000	720	11.50	6.30	114.28	1260.00	540.00
NG-3	0.48		1440	7.90	9.90	145.45	1980.00	540.00
S-1	0.50		5750	13.00	4.80	1197.91	960.00	-4790.00
S-2	1.00	11500	11500	10.10	7.70	1493.50	1540.00	-9960.00
S-3	2.00		23000	8.60	9.20	2500.00	1840.00	-21160.00
Loamy Soil								
CK (Control)	-	-	0	19.70	0	-	-	-
NG-1	0.12		360	16.00	3.70	97.30	740.00	380.00
NG-2	0.24	3000	720	12.80	6.90	104.35	1380.00	660.00
NG-3	0.48		1440	8.80	10.90	132.11	2180.00	740.00
S-1	0.50		5750	14.40	5.30	1084.90	1060.00	-4690.00
S-2	1.00	11500	11500	11.20	8.50	1352.94	1700.00	-9800.00
S-3	2.00		23000	9.60	10.10	2277.22	2020.00	-20980.00
Clay Loam Soil								
CK (Control)	-	-	-	23.70	-	-	-	-
NG-1	0.12		360	19.20	4.50	80.00	900.00	540.00
NG-2	0.24	3000	720	15.40	8.30	86.75	1660.00	940.00
NG-3	0.48		1440	10.60	13.10	109.92	2620.00	1180.00
S-1	0.50		5750	17.30	6.40	898.43	1280.00	-4470.00
S-2	1.00	11500	11500	13.40	10.30	1116.50	2060.00	-9440.00
S-3	2.00		23000	11.50	12.20	1885.24	2440.00	-20560.00

Note: The valuation parameter  $\lambda$  converts an ESP-point improvement into money via the threshold–slope yield–response model:  $\lambda = (s/100) \times Y \times P$ , where  $s$  is the percent yield change per 1 ESP point above the crop-specific ESP threshold,  $Y$  is baseline yield (t ha<sup>-1</sup>), and  $P$  is the farm-gate price (EGP t<sup>-1</sup>). We adopt  $\lambda = 200$  EGP ha<sup>-1</sup>.ESP<sup>-1</sup> as a conservative base case consistent with  $s \approx 0.2\%$  per ESP point and typical local yield  $\times$  price; robustness can be checked by sensitivity at  $\lambda = 500$  and 1000. Control rows are baselines ( $\Delta$ ESP = 0), thus cost per 1 ESP is undefined and benefit/net benefit are 0. References: Maas & Hoffman (1977); Richards (1954); Abrol et al., (1988); Gupta & Sharma (1990); Stinnett & Mullahy (1998).

#### D. Integrated chemical and physical responses of saline sodic soils to nano-gypsum and sulfur Soil Chemical Properties

The application of nano-gypsum (NG) and elemental sulfur (S) induced significant chemical improvements in saline–sodic soils, with consistent effects across amendments, soil textures, and incubation periods (Table 7).

##### pH and Carbonate Dissolution

Soil pH decreased from 8.89 in the control to 8.63–8.64 under NG-480 and S-2000 after six months. Progressive acidification over time (8.84  $\rightarrow$  8.58 between months 1 and 6) reflects both the immediate release of Ca<sup>2+</sup> from NG and the oxidative conversion of sulfur to H<sub>2</sub>SO<sub>4</sub>,

which dissolves native CaCO<sub>3</sub> and liberates additional Ca<sup>2+</sup>. Corresponding decreases in HCO<sub>3</sub><sup>-</sup> and CO<sub>3</sub><sup>2-</sup>, alongside concurrent increases in SO<sub>4</sub><sup>2-</sup> (3.20  $\rightarrow$  4.57 and 4.42 meq L<sup>-1</sup> under NG-480 and S-2000, respectively), confirm this pathway of carbonate dissolution and sulfur oxidation. These findings corroborate previous reports highlighting sulfur-induced acidification as a critical mechanism for mobilizing Ca<sup>2+</sup> in carbonate-rich saline–sodic soils (Tabatabai, 2005 and Malik *et al.*, 2021).

**Table 5. Cost–benefit of nano-gypsum and sulfur applications ( $\lambda = 500$  and  $\lambda = 1000$ ) in non-saline sodic soils (sandy, loam, clay loam) after 6 months**

Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate $\times$ price) (EGP/ha)	Estimated ESP, %	$\Delta$ ESP, % (ESP <sub>cont.</sub> – ESP <sub>treat.</sub> )	Cost per 1 % ESP decreased (EGP) (Total cost / $\Delta$ ESP)	Benefit (EGP/ha) at $\lambda=500$ (500 $\times$ $\Delta$ ESP)	Net Benefit (EGP/ha) at $\lambda=500$ (Benefit – Cost)	Benefit (EGP/ha) at $\lambda=1000$ (1000 $\times$ $\Delta$ ESP)	Net Benefit (EGP/ha) at $\lambda=1000$ (Benefit – Cost)
Sandy Soil										
CK (Control)	-	-	-	17.80	-	-	-	-	-	-
NG-1	0.12		360	14.40	3.40	105.88	1700.00	1340.00	3400.00	3040.00
NG-2	0.24	3000	720	11.50	6.30	114.28	3150.00	2430.00	6300.00	5580.00
NG-3	0.48		1440	7.90	9.90	145.45	4950.00	3510.00	9900.00	8460.00
S-1	0.50		5750	13.00	4.80	1197.91	2400.00	-3350.00	4800.00	-950.00
S-2	1.00		11500	10.10	7.70	1493.50	3850.00	-7650.00	7700.00	-3800.00
S-3	2.00	11500	23000	8.60	9.20	2500.00	4600.00	-	9200.00	-13800.00
								18400.00		
Loamy Soil										
CK (Control)	-	-	-	19.70	-	-	-	-	-	-
NG-1	0.12		360	16.00	3.70	97.30	1850.00	1490.00	3700.00	3340.00
NG-2	0.24	3000	720	12.80	6.90	104.35	3450.00	2730.00	6900.00	6180.00
NG-3	0.48		1440	8.80	10.90	132.11	5450.00	4010.00	10900.00	9460.00
S-1	0.50		5750	14.40	5.30	1084.90	2650.00	-3100.00	5300.00	-450.00
S-2	1.00		11500	11.20	8.50	1352.94	4250.00	-7250.00	8500.00	-3000.00
S-3	2.00	11500	23000	9.60	10.10	2277.22	5050.00	-	10100.00	-12900.00
								17950.00		
Clay Loam Soil										
CK (Control)	-	-	-	23.70	-	-	-	-	-	-
NG-1	0.12		360	19.20	4.50	80.00	2250.00	1890.00	4500.00	4140.00
NG-2	0.24	3000	720	15.40	8.30	86.75	4150.00	3430.00	8300.00	7580.00
NG-3	0.48		1440	10.60	13.10	109.92	6550.00	5110.00	13100.00	11660.00
S-1	0.50		5750	17.30	6.40	898.43	3200.00	-2550.00	6400.00	650.00
S-2	1.00		11500	13.40	10.30	1116.50	5150.00	-6350.00	10300.00	-1200.00
S-3	2.00	11500	23000	11.50	12.20	1885.24	6100.00	-	12200.00	-10800.00
								16900.00		

Note: The valuation parameter  $\lambda$  converts an ESP-point improvement into money via the threshold–slope yield–response model:  $\lambda = (s/100) \times Y \times P$ , where  $s$  is the percent yield change per 1 ESP point above the crop-specific ESP threshold,  $Y$  is baseline yield (t ha<sup>-1</sup>), and  $P$  is the farm-gate price (EGP t<sup>-1</sup>). We adopt  $\lambda = 200$  EGP ha<sup>-1</sup>.ESP<sup>-1</sup> as a conservative base case consistent with  $s \approx 0.2\%$  per ESP point and typical local yield  $\times$  price; robustness can be checked by sensitivity at  $\lambda = 500$  and 1000. Control rows are baselines ( $\Delta$ ESP = 0), thus cost per 1 ESP is undefined and benefit/net benefit are 0. References: Maas & Hoffman (1977); Richards (1954); Abrol et al., (1988); Gupta & Sharma (1990); Stinnett & Mullahy (1998).

### Salinity and Sodicty Amelioration

Electrical conductivity of the saturation extract (ECe) declined significantly, with reductions of ~20–30% under higher amendment rates (16.39  $\rightarrow$  12.64 dS m<sup>-1</sup> in S-2000). More importantly, sodicty indices showed consistent improvement: exchangeable Na<sup>+</sup> decreased from 6.77 to 5.32 cmolc kg<sup>-1</sup>, ESP from 32.3% to 25.0%, and SAR from 25.8 to 20.0. Temporal effects were particularly pronounced: after six months, ESP declined to 20.9% and SAR to 16.7, compared with

31.4% and 24.5%, respectively, at one month. These improvements reflect sustained Na<sup>+</sup>–Ca<sup>2+</sup> exchange and leaching facilitated by amendment-derived Ca<sup>2+</sup> and enhanced ionic mobility under leaching regimes. Comparable amelioration has been reported in long-term gypsum and sulfur studies, which document significant reductions in sodicty while alleviating osmotic stress (Bello *et al.*, 2021 and Singh *et al.*, 2023).

**Table 6. Non-saline sodic soils (6 months): Incremental cost-effectiveness of structural factor (F) improvement—cost (EGP) per 0.01 F\* gained vs control by treatment and texture, with dominance classification**

Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate × price) (EGP/ha)	Computed F*	$\Delta F^*$ ( $F^*_{\text{treatment}} - F^*_{\text{control}}$ )	Cost per 0.01 % F* increased (EGP) (Total cost / $\Delta F^* / 0.01$ )	Status	Notes
Sandy Soil								
CK (Control)		-	-	1.00	-	-	Baseline control	-
NG-1	0.12		360	1.58	0.58	6.21	Non-dominated (frontier)	-
NG-2	0.24	3000	720	2.08	1.08	6.67	Non-dominated (frontier); Dominant (vs some)	Dominates S-1
NG-3	0.48		1440	2.87	1.87	7.70	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 and S-2
S-1	0.50		5750	1.92	0.92	62.50	Dominated	Dominated by NG-2 and NG-3
S-2	1.00	11500	11500	2.46	1.46	78.76	Dominated	Dominated by NG-3
S-3	2.00		23000	3.00	2.00	115.00	Non-dominated (frontier)	-
Loamy Soil								
CK (Control)		-	0	1.00	-	-	Baseline control	-
NG-1	0.12		360	1.50	0.50	7.20	Non-dominated (frontier)	-
NG-2	0.24	3000	720	1.99	0.99	7.27	Non-dominated (frontier); Dominant (vs some)	Dominates S-1
NG-3	0.48		1440	2.75	1.75	8.23	Non-dominated (frontier); Dominant (vs some)	Dominates S-2 (also Dominates S-1)
S-1	0.50		5750	1.79	0.79	72.78	Dominated	Dominated by NG-2 (also by NG-3)
S-2	1.00	11500	11500	2.31	1.31	87.78	Dominated	Dominated by NG-3
S-3	2.00		23000	2.81	1.81	127.07	Non-dominated (frontier)	-
Clay Loam Soil								
CK (Control)		-	-	1.00	-	-	Baseline control	-
NG-1	0.12		360	1.53	0.53	6.79	Non-dominated (frontier)	-
NG-2	0.24	3000	720	2.03	1.03	6.99	Non-dominated (frontier); Dominant (vs some)	Dominates S-1
NG-3	0.48		1440	2.83	1.83	7.87	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 and S-2
S-1	0.50		5750	1.82	0.82	70.12	Dominated	Dominated by NG-2 and NG-3
S-2	1.00	11500	11500	2.36	1.36	84.56	Dominated	Dominated by NG-3
S-3	2.00		23000	2.88	1.88	122.34	Non-dominated (frontier)	-

Note: Status is determined within each texture: Dominant (vs some) = cheaper and more effective than at least one alternative; Dominated = more expensive and less (or no more) effective → excluded; non-dominated (frontier) = option remaining after removing dominated. References: (Drummond et al., 2015); and (NICE, 2025).

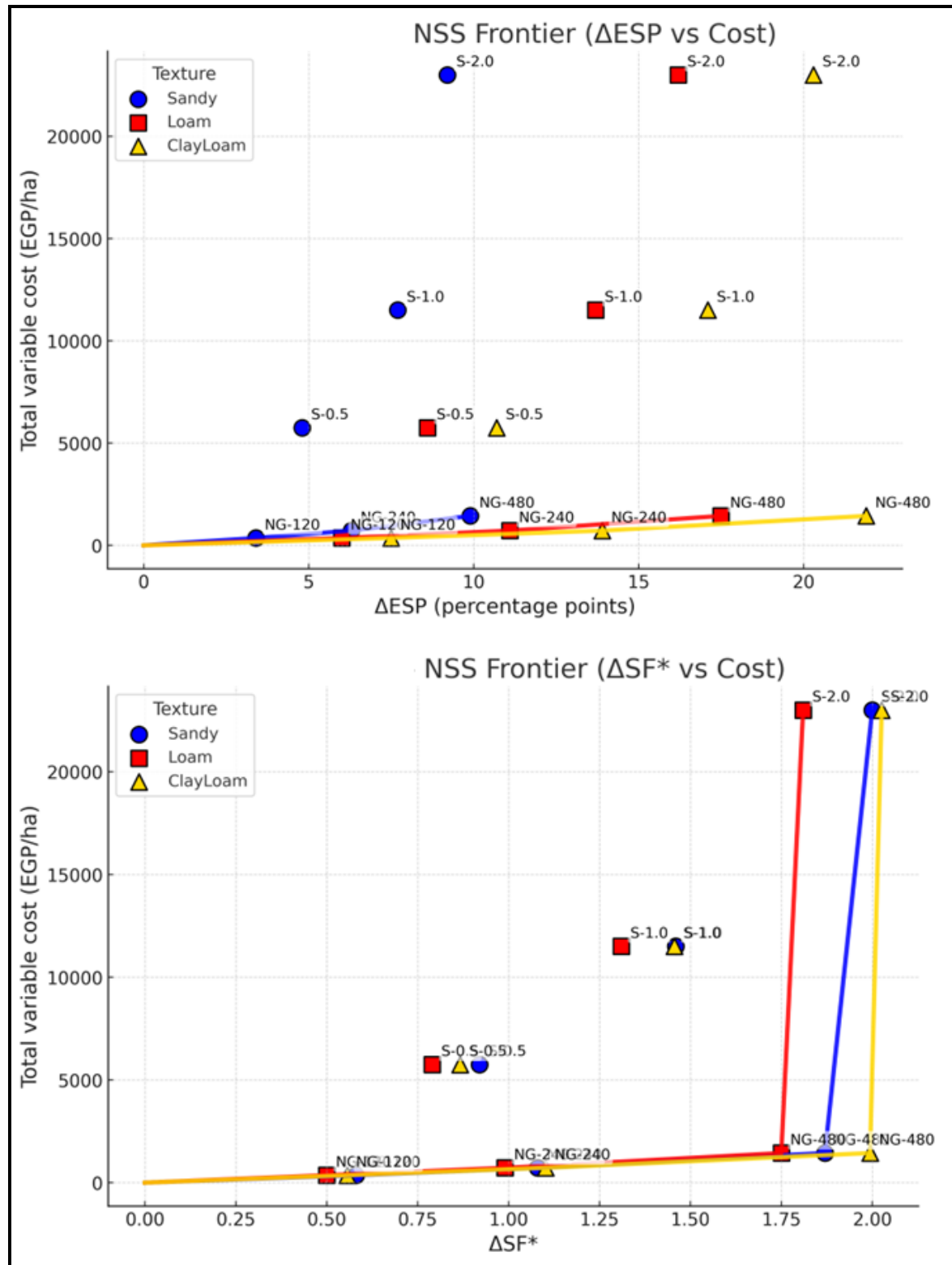


Fig. 7. Cost-effectiveness frontiers for non-saline-sodic (SS) soils at six months, showing  $\Delta ESP$  and  $\Delta SF^*$  relative to total variable cost (EGP ha<sup>-1</sup>), by soil texture

### Texture-Specific Responses

Reclamation efficiency was strongly influenced by soil texture. Sandy soils exhibited the most rapid response, with ESP averaging 21% and SAR 17.1 after treatment, compared with 33.1% and 27.0 in clay loam. Loam soils showed intermediate improvements. The slower amelioration in fine-textured, carbonate-rich soils can be attributed to their higher  $\text{CaCO}_3$  content and cation exchange capacity (CEC), which buffer pH decline and delay  $\text{Na}^+$  displacement. These outcomes agree with prior findings that fine-textured soils require higher amendment rates or longer incubation periods to achieve comparable sodicity reductions (Elgala *et al.*, 2021 and Rezapour *et al.*, 2023).

### Integrated Mechanisms

Taken together, the results indicate that NG provides an immediate source of soluble  $\text{Ca}^{2+}$ , initiating rapid  $\text{Na}^+$  exchange and double-layer collapse, while S ensures gradual acidification and sustained  $\text{Ca}^{2+}$  release through continuous carbonate dissolution. The

significant amendment  $\times$  texture  $\times$  period interactions (Table 7) emphasize the combined influence of amendment chemistry, soil buffering capacity, and time on reclamation efficiency. This dual mechanism explains the sequential improvements in pH, ECe, exchangeable  $\text{Na}^+$ , ESP, and SAR. These chemical shifts establish the foundation for subsequent physical property improvement, including enhancements in Ks, AI, and  $\text{SF}^*$  (Table 8; Figs. 8–11).

Overall, the results demonstrate that saline–sodic soils respond more slowly and require greater amendment effort than non-saline sodic soils, owing to their higher soluble salt concentrations and greater  $\text{CaCO}_3$  buffering capacity. Nevertheless, the combined application of NG and S provides a robust reclamation pathway by simultaneously addressing sodicity ( $\text{Na}^+$  saturation) and salinity (ECe), both of which must be mitigated to restore soil productivity.

**Table 7. ANOVA significance levels and mean values of soil chemical properties in saline–sodic soils amended with nano-gypsum and sulfur**

Variation	pH	ECe (dS m <sup>-1</sup> )	CEC (cmolc kg <sup>-1</sup> )	Exch. Na (cmolc kg <sup>-1</sup> )	ESP (%)	SAR	$\text{HCO}_3^-$	$\text{CO}_3^{2-}$ meq/L	$\text{SO}_4^{2-}$
<b>Amendments (A)</b>	***	***	NS	***	***	***	**	**	**
– Control	8.89	16.39	20.0	6.77	32.3	25.8	8.29	3.17	3.20
– NG-120	8.76	15.69	20.0	6.56	30.9	24.8	7.86	2.97	3.24
– NG-240	8.70	14.93	20.0	6.21	28.6	22.8	7.62	2.90	3.81
– NG-480	8.63	13.39	20.0	5.50	25.0	20.1	7.35	2.81	4.57
– S-500	8.74	14.97	20.0	6.20	29.7	23.6	7.80	2.95	3.57
– S-1000	8.69	13.63	20.0	5.67	27.0	21.5	7.43	2.89	4.10
– S-2000	8.64	12.64	20.0	5.32	25.0	20.0	7.20	2.84	4.42
LSD <sub>0.05</sub> (A)	0.03	0.10	0.40	0.07	0.60	0.60	0.20	0.05	0.90
<b>Texture (T)</b>	***	**	NS	***	***	***	**	**	**
– Sandy	8.70	8.28	13.0	2.84	21.0	17.1	7.53	2.89	4.10
– Loam	8.71	12.69	20.0	5.05	25.7	20.6	7.56	2.87	4.07
– Clay loam	8.81	18.40	27.0	8.83	33.1	27.0	7.75	2.92	4.18
LSD <sub>0.05</sub> (T)	0.03	0.10	0.40	0.07	0.60	0.60	0.20	0.05	0.90
<b>Period (P)</b>	***	**	NS	***	***	***	**	**	**
– 1 month	8.84	15.39	20.0	6.71	31.4	24.5	7.94	2.99	3.30
– 3 months	8.70	12.48	20.0	5.41	25.5	20.6	7.43	2.85	4.28
– 6 months	8.58	9.81	20.0	4.33	20.9	16.7	6.97	2.73	5.05
LSD <sub>0.05</sub> (P)	0.03	0.10	0.40	0.07	0.60	0.60	0.20	0.05	0.90
<b>Interactions</b>									
A $\times$ T	**	*	NS	**	**	**	*	*	*
A $\times$ P	**	*	NS	**	**	**	*	*	*
T $\times$ P	**	*	NS	**	**	**	*	*	*
A $\times$ T $\times$ P	*	*	NS	*	**	*	*	*	*

Note: The table presents ANOVA results, including main effects (amendments, textures, and periods) and their interactions for soil chemical properties. Mean values are shown for seven treatments, three soil textures, and three incubation periods. LSD<sub>0.05</sub> values are reported separately for amendments (A), textures (T), and periods (P). NS = non-significant; \* =  $P \leq 0.05$ ; \*\* =  $P \leq 0.01$ ; \*\*\* =  $P \leq 0.001$ .

### Soil physical properties

The application of nano-gypsum (NG) and elemental sulfur (S) produced significant improvements in soil physical conditions, with consistent effects across amendments, textures, and incubation periods (Table 8). These changes reflect the downstream impact of sodicity reduction, whereby chemical amelioration enhanced aggregation, water transmission, and structural resilience.

### Bulk Density and Hydraulic Conductivity

Bulk density (Bd) remained statistically unchanged (mean  $\approx 1.43 \text{ g cm}^{-3}$  across treatments), indicating that six months of incubation was insufficient to alter packing density at the bulk scale. In contrast, saturated hydraulic conductivity (Ks) responded strongly, rising from  $0.93 \text{ cm h}^{-1}$  in the control to  $1.73\text{--}1.75 \text{ cm h}^{-1}$  under NG-480 and S-2000 ( $\approx 85\text{--}90\%$  increase). Temporal trends showed Ks increasing steadily from  $1.16 \text{ cm h}^{-1}$  at one month to  $1.90 \text{ cm h}^{-1}$  after six months, confirming that sodicity relief progressively improved pore continuity. Similar gains in Ks following gypsum and sulfur treatments have been attributed to ESP reduction and clay flocculation (Green *et al.*, 2023 and Xiao *et al.*, 2025).

### Water Retention and Available Water

Field water retention parameters improved modestly under NG and S. Water content at  $-33 \text{ kPa}$  ( $\theta_{33}$ ) increased from 22.53% (control) to 23.40% under S-2000, while permanent wilting point at  $-1500 \text{ kPa}$  ( $\theta_{1500}$ ) increased slightly ( $9.70 \rightarrow 9.77\%$ ). Consequently, available water (AW) increased from 12.83% in the control to 13.63% under S-2000. These modest increases suggest enhanced mesoporosity without substantial change in micropore domains, consistent with the early stages of structural recovery.

### Aggregate Stability and Structural Indices

Amendments markedly improved stability indicators. The aggregate index (AI) rose from 35.0 in the control to 54.3 under NG-480 and 58.8 under S-2000, representing  $\sim 60\%$  enhancement. Similarly, the structure factor (SF) nearly doubled from 6.83 to 13.5, while the normalized structure factor (SF\*) also increased from 1.00  $\rightarrow 1.64\text{--}1.72$ , indicating clear gains in structural stability. Over time, AI increased from 39.3 at one month to 53.9 at six months, while SF nearly tripled ( $8.6 \rightarrow 19.0$ ), and SF\* rose to 1.95, confirming cumulative benefits of prolonged incubation (Fig. 11).

### Texture-Specific Responses

Textural contrasts shaped the magnitude of improvement. Sandy soils exhibited the highest Ks ( $2.42 \text{ cm h}^{-1}$ ) but low water retention ( $\theta_{-33} = 16.8\%$ ,  $\theta_{-1500} = 3.57\%$ ), limiting AW (13.23%). Clay loam soils

showed high retention ( $\theta_{-33} = 31.6\%$ ,  $\theta_{-1500} = 14.87\%$ ) but severely constrained Ks ( $0.27 \text{ cm h}^{-1}$ ). Loam soils occupied an intermediate position. These results highlight that while chemical amelioration alleviates sodicity across all textures, the expression of physical recovery is strongly conditioned by intrinsic soil properties (Rezapour *et al.*, 2023).

### Integrated Mechanisms

The improvements in Ks, AW, and stability indices can be directly linked to chemical amelioration. Rapid  $\text{Ca}^{2+}$  supply from NG and sustained acidification from S lowered ESP and SAR, promoting flocculation, reconnection of blocked pores, and stabilization of soil aggregates. Significant amendment  $\times$  texture  $\times$  period interactions (Table 8) emphasize that effectiveness depends on the combined influence of amendment chemistry, soil buffering, and time.

Overall, these findings confirm that NG and S not only correct the chemical constraints of saline-sodic soils but also drive physical rehabilitation. Enhanced infiltration, improved structural stability, and modest gains in water availability provide the functional basis for higher soil productivity and resilience under cropping systems.

### E. Six-Month Cost-Benefit and Cost-Effectiveness of Nano-Gypsum vs. Elemental Sulfur in Saline-Sodic Soils

Over a six-month period in saline-sodic (SS) soils, nano-gypsum (NG) consistently surpasses elemental sulfur ( $\text{S}^0$ ) in both cost-benefit and cost-effectiveness across sandy, loam, and clay-loam textures, with the greatest advantage observed in coarse soils where hydraulic improvement and ESP decline accrue fastest.

Using a conservative valuation parameter for ESP reduction ( $\lambda = 200 \text{ EGP ha}^{-1}$  per ESP point), all NG rates produce positive net benefits (NB) by month 6, whereas all  $\text{S}^0$  rates remain negative due to higher costs and slower oxidation. For example, in sandy SS soil, NB increases from +620 to +1,100  $\text{EGP ha}^{-1}$  for NG-1  $\rightarrow$  NG-2, with NG-3 still positive (+880  $\text{EGP ha}^{-1}$ ). By contrast, S-1, S-2, and S-3 yield  $-5,210$ ,  $-9,940$ , and  $-20,940 \text{ EGP ha}^{-1}$ , respectively (Table 9). Similar trends are observed in loam (NG: +840 to +2,060;  $\text{S}^0$ : all negative) and clay-loam (NG: +1,140 to +2,940;  $\text{S}^0$ : all negative). These differences reflect larger  $\Delta\text{ESP}$  achieved at modest NG costs, versus delayed  $\text{S}^0$  oxidation and high material outlays.

Incremental cost-effectiveness analysis (ICER) confirms this ranking whether effectiveness is measured as ESP reduction (Table 10) or normalized structure factor, SF\* (Table 11). In sandy soils, NG-1 and NG-3 lie on the efficiency frontier, NG-2 is weakly

dominated, and S-1/S-2 are strictly dominated. S-3 occasionally appears on the frontier but only at prohibitively high ICERs. The same dominance structure recurs in loam and clay-loam: NG consistently efficient, S-1/S-2 dominated—more expensive and less effective than NG options—, and S-3 viable only at ICERs one to two orders of magnitude higher.

Mechanistically, NG's advantage reflects classical sodic-soil remediation: soluble  $\text{Ca}^{2+}$  rapidly displaces  $\text{Na}^+$ , collapses diffuse double layers, promotes flocculation, reopening macropores, and enhances salt/Na leaching. These processes are texture-sensitive and disproportionately benefit coarse soils with higher intrinsic permeability (Richards, 1954 and NRCS CPS-333, 2024). By contrast,  $\text{S}^0$  requires microbial oxidation to  $\text{H}_2\text{SO}_4$  before  $\text{CaCO}_3$  dissolution can supply  $\text{Ca}^{2+}$ , delaying benefits by weeks–months (Degryse *et al.*, 2016).

Methodologically, the study applies standard cost-effectiveness rules: net-benefit framework ( $\text{NB} = \lambda \times \Delta\text{ESP} - \Delta\text{Cost}$ ), ICER computation after eliminating dominated options, and  $\lambda$  grounded in a threshold–slope model linking ESP to yield loss (Drummond *et al.*, 2015

and NICE HTA Manual, 2022). Within a six-month reclamation window in SS soils, NG at modest–moderate rates ( $0.12\text{--}0.48 \text{ t ha}^{-1}$ ) is the economically efficient choice across textures (Fig. 12).  $\text{S}^0$  becomes attractive only under longer periods and/or higher  $\lambda$  values (high-value, salt-sensitive crops) where delayed acidification benefits can offset high costs.

#### F- Comparative Reclamation in Non-Saline Sodic and Saline-Sodic Soils: Nano-Gypsum vs Elemental Sulfur Across Textures

In non-saline sodic (NSS) and saline-sodic (SS) soils, the operational reclamation targets converge on lowering ESP below  $\approx 15\%$  (desodification), while SS soils additionally require reducing ECe below  $\approx 4 \text{ dS m}^{-1}$  to resolve salinity; the Siwa field dataset and classical guidance are consistent on these thresholds and on the Ca-based sequencing of amendment plus leaching (gypsum or  $\text{S}^0 \rightarrow \text{H}_2\text{SO}_4 \rightarrow \text{CaCO}_3$  dissolution  $\rightarrow \text{Ca}^{2+}$  supply  $\rightarrow \text{Na}^+$  displacement  $\rightarrow$  leaching).

**Table 8. ANOVA significance and mean values of soil physical properties in saline sodic soils amended with nano-gypsum and sulfur**

Variation	Bd( $\text{g cm}^{-3}$ )	Ks( $\text{cm h}^{-1}$ )	0-33(%)	0-1500(%)	AW(%)	AI	SF	SF*
Amendments(A)	***	***	**	**	***	***	**	**
– Control	1.43	0.93	22.53	9.70	12.83	35.0	6.83	1.00
– NG-120	1.43	1.12	22.97	9.73	13.23	39.5	8.23	1.14
– NG-240	1.43	1.28	23.10	9.73	13.37	43.9	9.60	1.27
– NG-480	1.43	1.73	23.27	9.73	13.53	54.3	12.55	1.64
– S-500	1.43	1.10	23.27	9.73	13.53	39.5	8.12	1.12
– S-1000	1.43	1.29	23.33	9.77	13.57	47.9	10.63	1.34
– S-2000	1.43	1.75	23.40	9.77	13.63	58.8	13.50	1.72
LSD <sub>0.05</sub> (A)	0.04	0.25	0.30	0.30	0.30	17.99	4.13	0.62
Texture (T)	***	***	***	***	***	***	***	***
– Sandy	1.58	2.42	16.8	3.57	13.23	49.0	16.3	1.46
– Loam	1.42	0.96	25.0	9.87	15.13	49.6	11.5	1.59
– Clay loam	1.30	0.27	31.6	14.87	16.73	48.4	2.9	1.63
LSD <sub>0.05</sub> (T)	0.04	0.25	0.30	0.30	0.30	17.99	4.13	0.62
Period (P)	***	***	***	**	***	***	***	***
– 1 month	1.43	1.16	21.8	9.8	12.0	39.3	8.6	1.14
– 3 months	1.43	1.60	22.7	9.8	12.9	45.8	13.1	1.53
– 6 months	1.43	1.90	23.5	9.9	13.6	53.9	19.0	1.95
LSD <sub>0.05</sub> (P)	0.04	0.25	0.30	0.30	0.30	17.99	4.13	0.62
Interactions								
A $\times$ T	**	**	*	*	**	**	*	*
A $\times$ P	**	**	*	*	**	**	*	*
T $\times$ P	**	**	*	*	**	**	*	*
A $\times$ T $\times$ P	*	*	NS	NS	*	*	*	*

Note: The table summarizes ANOVA results, including main effects (amendments, textures, and periods) and their interactions for soil physical properties. Mean values are presented for seven treatments, three soil textures, and three incubation periods. LSD<sub>0.05</sub> values are reported separately for amendments (A), textures (T), and periods (P). NS = non-significant; \* =  $P \leq 0.05$ ; \*\* =  $P \leq 0.01$ ; \*\*\* =  $P \leq 0.001$ .

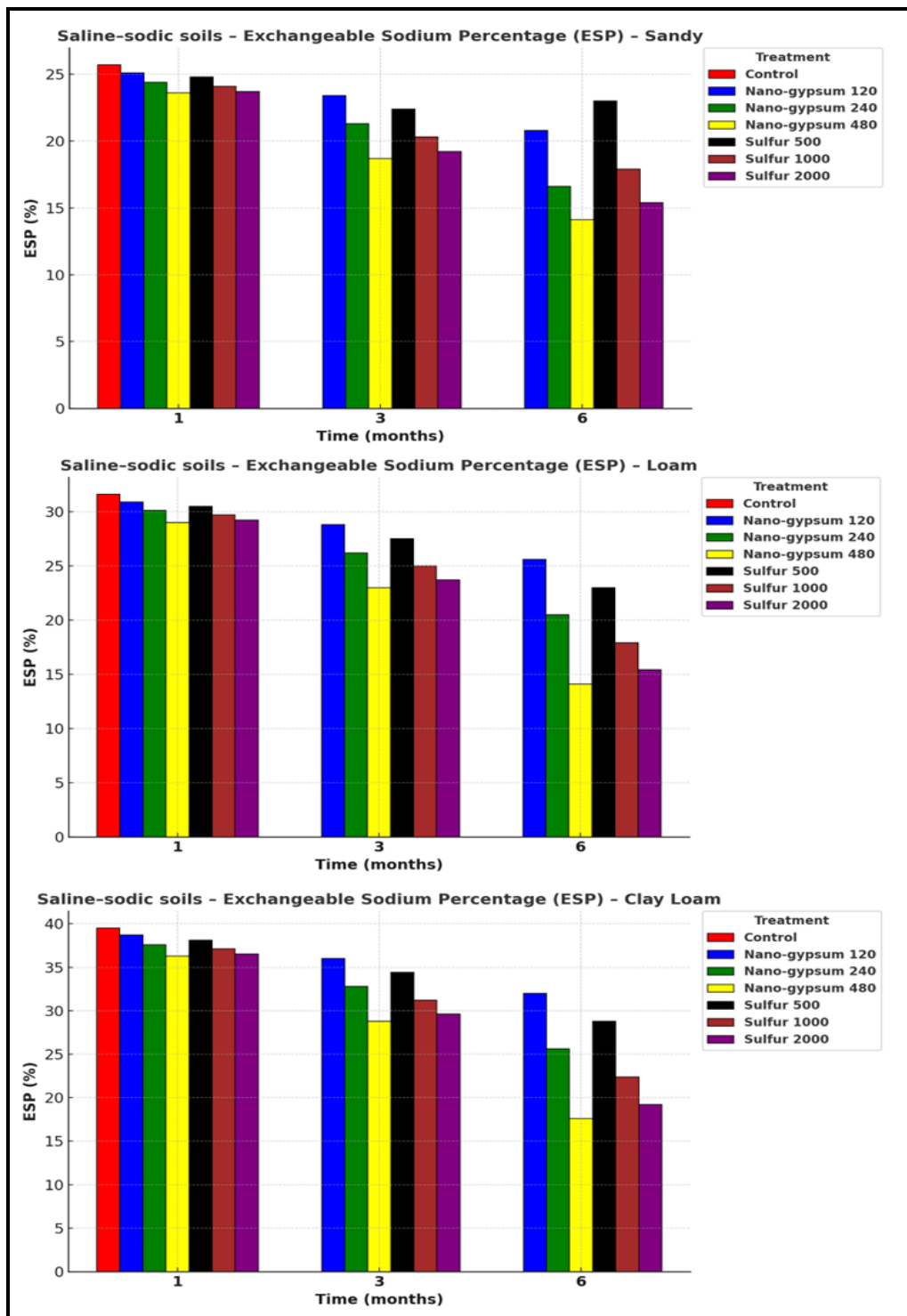


Fig. 8. Effect of nano-gypsum and elemental sulfur amendments on soil exchangeable sodium percentage (ESP, %) at three-time intervals (1, 3, and 6 months after application) in saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

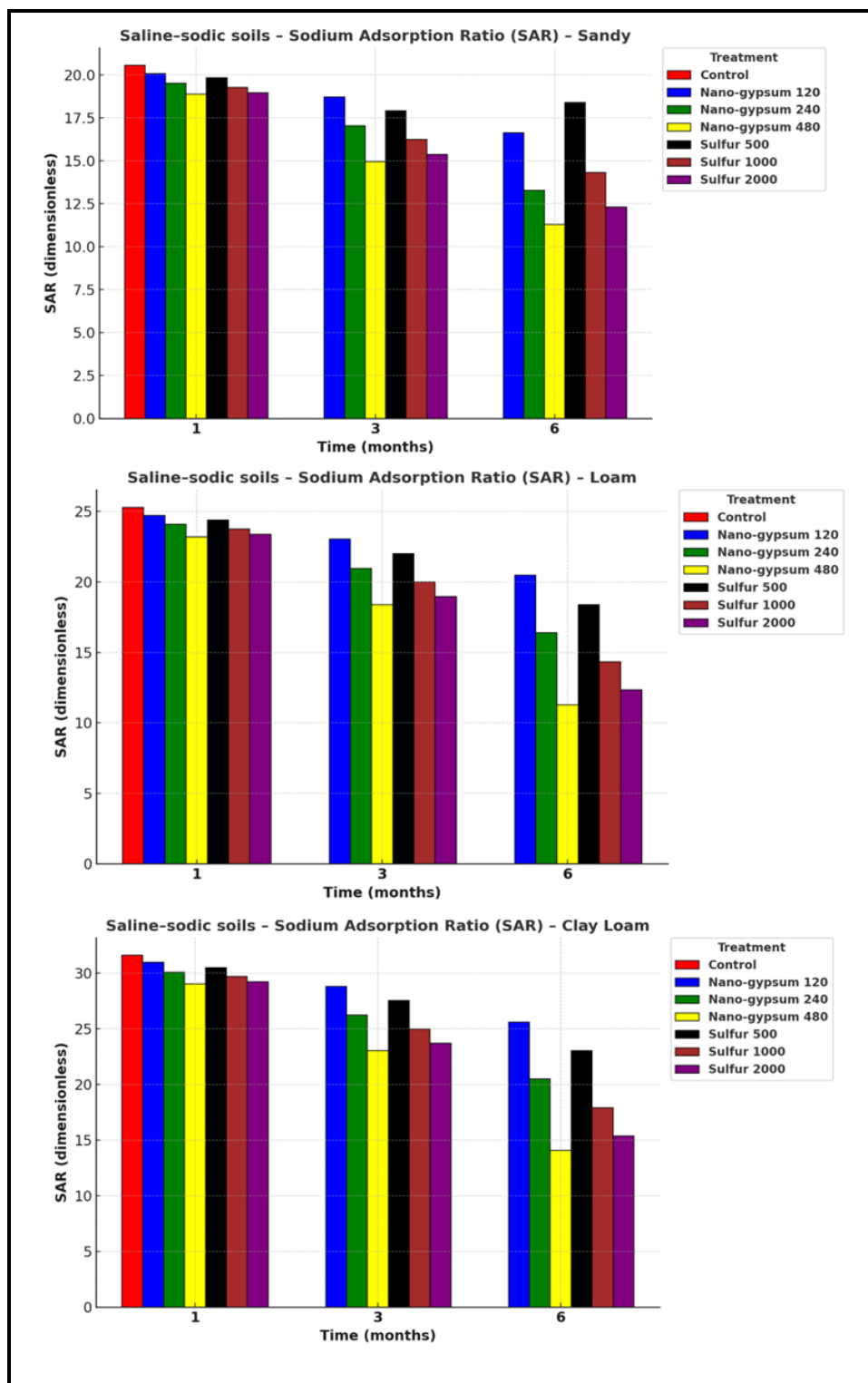


Fig. 9. Effect of nano-gypsum and elemental sulfur amendments on Sodium adsorption ratio (SAR) at three-time intervals (1, 3, and 6 months after application) in saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

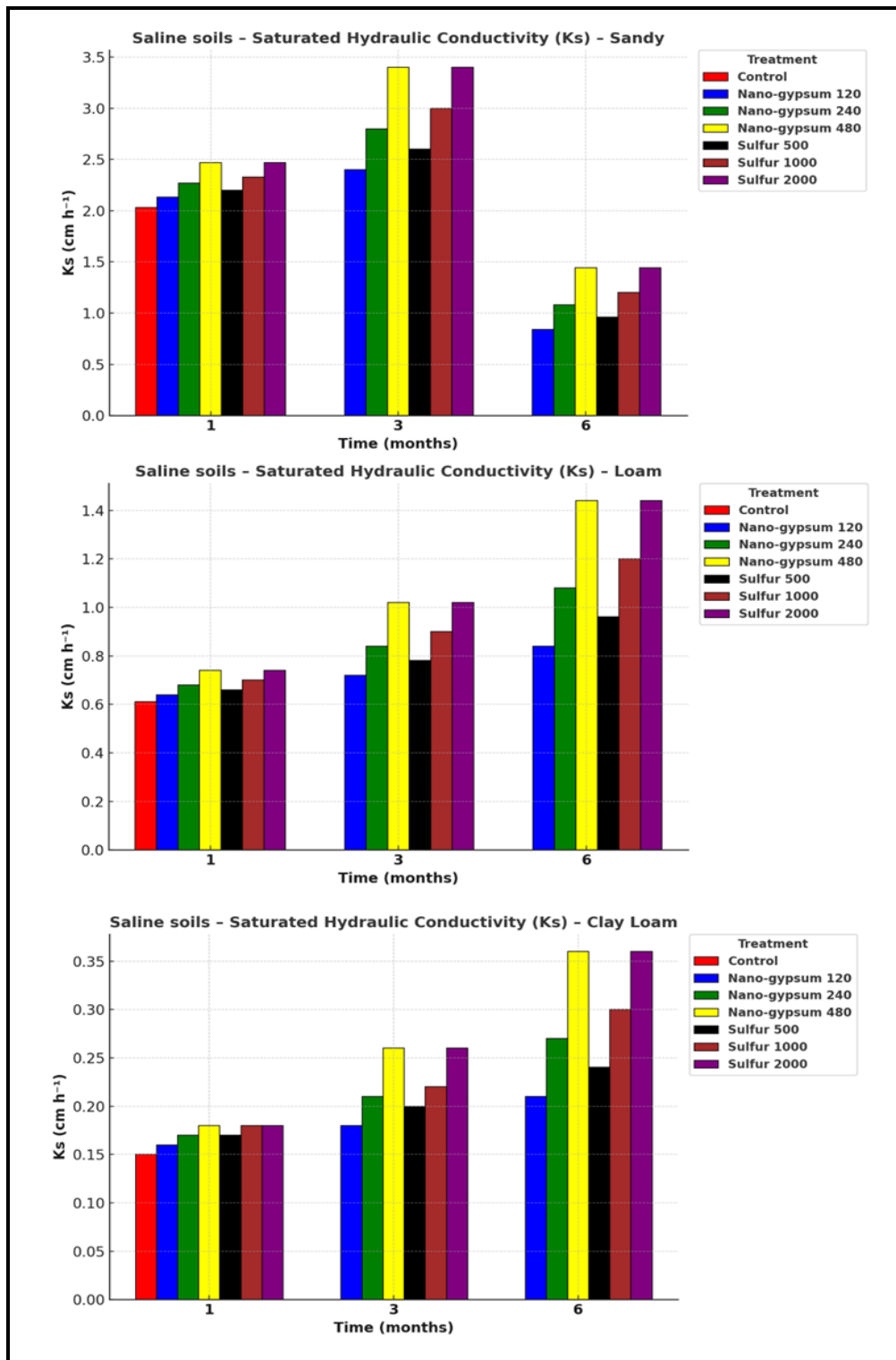


Fig. 10. Effect of nano-gypsum and elemental sulfur amendments on Saturated hydraulic conductivity ( $K_s$ ,  $\text{cm h}^{-1}$ ) at three-time intervals (1, 3, and 6 months after application) in saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

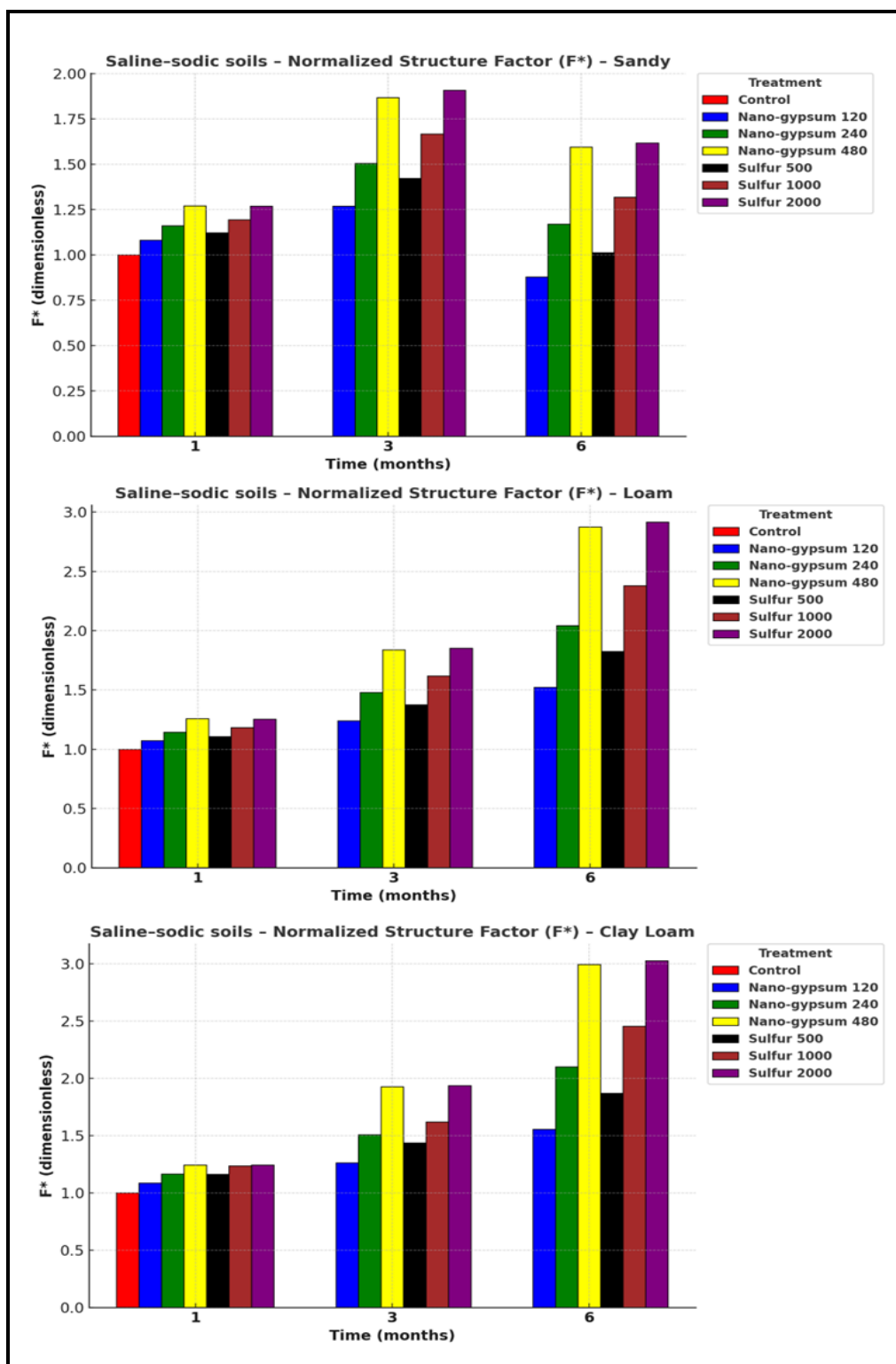


Fig. 11. Effect of nano-gypsum and elemental sulfur amendments on normalized structure factor (SF\*) at three-time intervals (1, 3, and 6 months after application) in saline sodic soils of three textures:(a) sandy, (b) loam, (c) clay loam

**Table 9. Cost–benefit of nano-gypsum and sulfur applications ( $\lambda = 200$ ) in saline sodic soils (sandy, loam, clay loam) after 6 months**

Loam/after 6 months								
Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate× price) (EGP/ha)	Estimated ESP, %	ΔESP, % (ESP <sub>contro</sub> – ESP <sub>treatment</sub> )	Cost per 1 % ESP decreased (EGP) (Total cost/ ΔESP)	Benefit (EGP/ha) at λ=200 (200×ΔESP)	Net Benefit (EGP/ha) at λ=200 (Benefit-Cost)
Sandy Soil								
CK (Control) -	-	-	-	25.70	-	-	-	-
NG-1	0.12	3000	360	20.80	4.90	73.47	980.00	620.00
NG-2	0.24		720	16.60	9.10	79.12	1820.00	1100.00
NG-3	0.48		1440	14.10	11.60	124.14	2320.00	880.00
S-1	0.50	11500	5750	23.00	2.70	2129.63	540.00	-5210.00
S-2	1.00		11500	17.90	7.80	1474.36	1560.00	-9940.00
S-3	2.00		23000	15.40	10.30	2233.00	2060.00	-20940.00
Loamy Soil								
CK (Control) -	-	-	0	31.60	-	-	-	-
NG-1	0.12	3000	360	25.60	6.00	60.00	1200.00	840.00
NG-2	0.24		720	20.50	11.10	64.86	2220.00	1500.00
NG-3	0.48		1440	14.10	17.50	82.28	3500.00	2060.00
S-1	0.50	11500	5750	23.00	8.60	668.60	1720.00	-4030.00
S-2	1.00		11500	17.90	13.70	839.42	2740.00	-8760.00
S-3	2.00		23000	15.40	16.20	1419.75	3240.00	-19760.00
Clay Loam Soil								
CK (Control) -	-	-	-	39.50	-	-	-	-
NG-1	0.12	3000	360	32.00	7.50	48.00	1500.00	1140.00
NG-2	0.24		720	25.60	13.90	51.80	2780.00	2060.00
NG-3	0.48		1440	17.60	21.90	65.75	4380.00	2940.00
S-1	0.50	11500	5750	28.80	10.70	537.38	2140.00	-3610.00
S-2	1.00		11500	22.40	17.10	672.51	3420.00	-8080.00
S-3	2.00		23000	19.20	20.30	1133.00	4060.00	-18940.00

Note: The valuation parameter  $\lambda$  converts an ESP-point improvement into money via the threshold–slope yield-response model:  $\lambda = (s/100) \times Y \times P$ , where  $s$  is the percent yield change per 1 ESP point above the crop-specific ESP threshold,  $Y$  is baseline yield (t ha<sup>-1</sup>), and  $P$  is the farm-gate price (EGP t<sup>-1</sup>). We adopt  $\lambda = 200$  EGP ha<sup>-1</sup>ESP<sup>-1</sup> as a conservative base case consistent with  $s \approx 0.2\%$  per ESP point and typical local yield  $\times$  price; robustness can be checked by sensitivity at  $\lambda = 500$  and 1000. Control rows are baselines ( $\Delta$ ESP = 0), thus cost per 1 ESP is undefined and benefit/net benefit are 0. References: Maas & Hoffman (1977); Richards (1954); Abrol *et al.* (1988); Gupta & Sharma (1990); Stinnett & Mullahy (1998).

In NSS soils, both nano-gypsum (NG) and elemental sulfur (S<sup>0</sup>) drove rapid, texture-modulated chemical improvement over six months, with NG furnishing immediately soluble Ca<sup>2+</sup> and S<sup>0</sup> acting via biological oxidation and CaCO<sub>3</sub> dissolution; sandy NSS ( $\approx 8\%$  CaCO<sub>3</sub>) decrease from ESP 17.8% to 7.9% and SAR 12.7  $\rightarrow$  6.7 at NG-480 (S-2000: ESP 8.6%, SAR 7.2) with Ece also declining to  $\sim 1.55$ – $1.36$  dS m<sup>-1</sup>, loam NSS ( $\approx 15\%$  CaCO<sub>3</sub>) from ESP 19.7% to 8.8% (S-2000: 9.6%) and SAR 13.8  $\rightarrow$  7.3 (7.8), and clay-loam NSS ( $\approx 28\%$  CaCO<sub>3</sub>) from ESP 23.7% to 10.6% (S-2000: 11.5%) and SAR 16.2  $\rightarrow$  8.4 (8.9), all meeting the non-sodic ESP criterion by month 6 despite stronger carbonate buffering in finer textures.

Physically, NSS soils showed coherent structural improvement: Ks roughly doubled to 2.4–2.6 $\times$  across textures (sandy: 3.56  $\rightarrow$  8.40 cm h<sup>-1</sup>; loam: 1.02  $\rightarrow$  2.40 cm h<sup>-1</sup>; clay-loam: 0.25  $\rightarrow$  0.60 cm h<sup>-1</sup>), available water increased modestly ( $\sim 0.3$ – $0.6\%$  v/v), and composite

structure indices strengthened (sandy AI  $\approx 35 \rightarrow 84$ – $100$ ; SF\*  $\approx 1.0 \rightarrow \approx 3.0$  by month 6), reflecting Ca-mediated flocculation and pore reconnection; responses ranked sandy > loam > clay-loam in line with permeability and buffering differences. In saline sodic (SS) soils, chemical and physical dynamics tracked together but progressed more slowly and with sharper texture controls: sandy SS desodified fastest (ESP 25.7%  $\rightarrow$  14.1% at 6 months under NG-480; S-2000: 15.4%) and salinity dropped (Ece 9.83  $\rightarrow$  5.00 dS m<sup>-1</sup> with NG-480; 4.40 dS m<sup>-1</sup> with S-2000), whereas loam SS reached ESP  $\approx 14.1$ – $15.4\%$  but remained saline (Ece  $\approx 7.0$ – $8.0$  dS m<sup>-1</sup>), and clay-loam SS stayed sodic at six months (ESP  $\approx 17.6$ – $19.2\%$ ; Ece  $\approx 10.6$ – $12.0$  dS m<sup>-1</sup>), underscoring the combined constraints of higher CEC and CaCO<sub>3</sub> buffering on Ca<sup>2+</sup> activity and Na<sup>+</sup> displacement.

**Table 10. Cost–benefit of nano-gypsum and sulfur applications ( $\lambda = 500$  and  $\lambda = 1000$ ) in saline sodic soils (sandy, loam, clay loam) after 6 months**

(Sandy, loam, clay loam) after 6 months										
Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate × price) (EGP/ha)	Estimated ESP, %	ΔESP, % (ESP <sub>cont.</sub> – ESP <sub>treat.</sub> )	Cost per 1 % ESP decreased (EGP) (Total cost / ΔESP)	Benefit (EGP/ha) at λ=500 (500 × ΔESP)	Net Benefit (EGP/ha) at λ=500 (Benefit – Cost)	Benefit (EGP/ha) at λ=1000 (1000 × ΔESP)	Net Benefit (EGP/ha) at λ=1000 (Benefit – Cost)
Sandy Soil										
CK (Control)	-	-	-	25.70	-	-	-	-	-	-
NG-1	0.12	3000	360	20.80	4.90	73.47	2450.00	2090.00	4900.00	4540.00
NG-2	0.24		720	16.60	9.10	79.12	4550.00	3830.00	9100.00	8380.00
NG-3	0.48		1440	14.10	11.60	124.14	5800.00	4360.00	11600.00	10160.00
S-1	0.50		5750	23.00	2.70	2129.63	1350.00	-4400.00	2700.00	-3050.00
S-2	1.00		11500	17.90	7.80	1474.36	3900.00	-7600.00	7800.00	-3700.00
S-3	2.00		23000	15.40	10.30	2233.00	5150.00	-17850.00	10300.00	-12700.00
Loamy Soil										
CK (Control)	-	-	-	31.60	-	-	-	-	-	-
NG-1	0.12	3000	360	25.60	6.00	60.00	3000.00	2640.00	600.00	240.00
NG-2	0.24		720	20.50	11.10	64.86	5550.00	4830.00	11100.00	10380.00
NG-3	0.48		1440	14.10	17.50	82.28	8750.00	7310.00	17500.00	16060.00
S-1	0.50		5750	23.00	8.60	668.60	4300.00	-1450.00	8600.00	2850.00
S-2	1.00		11500	17.90	13.70	839.42	6850.00	-4650.00	13700.00	2200.00
S-3	2.00		23000	15.40	16.20	1419.75	8100.00	-14900	16200.00	-6800.00
Clay Loam Soil										
CK (Control)	-	-	-	39.50	-	-	-	-	-	-
NG-1	0.12	3000	360	32.00	7.50	48.00	3750.00	3390.00	7500.00	7140.00
NG-2	0.24		720	25.60	13.90	51.80	6950.00	6230.00	13900.00	13180.00
NG-3	0.48		1440	17.60	21.90	65.75	10950.00	9510.00	21900.00	20460.00
S-1	0.50		5750	28.80	10.70	537.38	5350.00	-400.00	10700.00	4950.00
S-2	1.00		11500	22.40	17.10	672.51	8550.00	-2950.00	17100.00	5600.00
S-3	2.00		23000	19.20	20.30	1133.00	10150.00	-12850.00	20300.00	-2700.00

Note: The valuation parameter  $\lambda$  converts an ESP-point improvement into money via the threshold–slope yield–response model:  $\lambda = (s/100) \times Y \times P$ , where  $s$  is the percent yield change per 1 ESP point above the crop-specific ESP threshold,  $Y$  is baseline yield (t ha<sup>-1</sup>), and  $P$  is the farm-gate price (EGP t<sup>-1</sup>). We adopt  $\lambda = 200$  EGP ha<sup>-1</sup>ESP<sup>-1</sup> as a conservative base case consistent with  $s \approx 0.2\%$  per ESP point and typical local yield  $\times$  price; robustness can be checked by sensitivity at  $\lambda = 500$  and 1000. Control rows are baselines ( $\Delta$ ESP = 0), thus cost per 1 ESP is undefined and benefit/net benefit are 0. References: Maas & Hoffman (1977); Richards (1954); Abrol *et al.* (1988); Gupta & Sharma (1990); Stinnett & Mullahy (1998).

Consistent with the chemical, SS physical responses were time-structured: in sandy SS, Ks increased early (control 2.03  $\rightarrow$  2.47 cm h<sup>-1</sup> at 1 month under NG-480/S-2000;  $\approx$ 3.40 cm h<sup>-1</sup> by month 3) and then moderated by month 6 ( $\sim$ 1.44 cm h<sup>-1</sup>) as mesopore storage consolidated; AI and SF\* nevertheless remained well above controls (AI  $\approx$ 67–68%; SF\*  $\approx$ 1.59–1.62), while loam SS exhibited steadier mid-term gains (Ks  $\approx$ 0.61  $\rightarrow$  1.02 cm h<sup>-1</sup> by month 3; SF\*  $\approx$ 1.84–1.85). Mechanistically, these patterns match the expected roles of amendments and electrolytes: NG outruns S<sup>0</sup> on six-month ESP/SAR decline via direct Ca<sup>2+</sup> supply, while S<sup>0</sup> often attains the lower ECe at a fixed period—

especially as texture tightens and carbonate pools enlarge—supporting a pragmatic sequence of early NG (to protect structure and accelerate Ca–Na exchange) followed by S<sup>0</sup> where longer-period alkalinity control and CaCO<sub>3</sub> dissolution are needed; sustained leaching/drainage remains essential in SS to cross the ECe < 4 dS m<sup>-1</sup> threshold. Finally, the study's own framing and the broader canon (USSS/FAO criteria; electrolyte/valence controls; nano-gypsum's high-surface-area kinetics vs biologically mediated S<sup>0</sup> oxidation) align with these field results and with regional reports that NG can produce rapid structural and agronomic gains under saline–sodic stress.

**Table 11. Saline sodic soils (6 months): Cost-effectiveness of structural factor (F)—cost (EGP) per 0.01 F gained vs control, by treatment and texture, with dominance classification. \*\***

Treatment	Rate (t/ha)	Price (EGP/t)	Total Cost (rate × price) (EGP/ha)	Computed F*	$\Delta F^*$ ( $F^*_{\text{treatment}} - F^*_{\text{control}}$ )	Cost per 0.01 % F* increased (EGP) (Total cost / $\Delta F^*/0.01$ )	Status	Notes
Sandy Soil								
CK (Control)		-	-	1.000	-	-	Baseline control	-
NG-1	0.12		360	1.170	0.170	21.17	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 (cheaper & more effective)
NG-2	0.24	3000	720	1.265	0.265	27.17	Extended dominated	Ruled out because ICER(NG-1→NG-2) > ICER(NG-2→NG-3); a mix of adjacent options is more efficient
NG-3	0.48		1440	1.594	0.594	24.24	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 and S-2
S-1	0.50		5750	1.012	0.012	4791.66	Dominated	Dominated by NG-1/NG-2/NG-3 (all cheaper & more effective)
S-2	1.00	11500	11500	1.319	0.319	360.50	Dominated	Dominated by NG-3 (cheaper & more effective)
S-3	2.00		23000	1.617	0.617	372.77	Non-dominated (frontier)	-
Loamy Soil								
CK (Control)		-	-	1.000	-	-	Baseline control	-
NG-1	0.12		360	1.522	0.522	6.89	Non-dominated (frontier)	-
NG-2	0.24	3000	720	2.040	1.040	6.92	Non-dominated (frontier); Dominant (vs some)	Dominates S-1
NG-3	0.48		1440	2.873	1.873	7.69	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 and S-2
S-1	0.50		5750	1.823	0.823	69.86	Dominated	Dominated by NG-2 (cheaper & more effective)
S-2	1.00	11500	11500	2.378	1.378	83.45	Dominated	Dominated by NG-3 (cheaper & more effective)
S-3	2.00		23000	2.915	1.915	120.10	Non-dominated (frontier)	-
Clay Loam Soil								
CK (Control)		-	-	1.000	-	-	Baseline control	-
NG-1	0.12		360	1.556	0.556	6.47	Non-dominated (frontier)	-
NG-2	0.24	3000	720	2.102	1.102	6.53	Non-dominated (frontier); Dominant (vs some)	Dominates S-1
NG-3	0.48		1440	2.994	1.994	7.22	Non-dominated (frontier); Dominant (vs some)	Dominates S-1 and S-2

S-1	0.50	5750	1.867	0.867	66.32	Dominated	Dominated by NG-2 and NG-3 (cheaper & more effective)
S-2	1.00	11500	2.456	1.456	78.98	Dominated	Dominated by NG-3 (cheaper & more effective)
S-3	2.00	23000	3.026	2.026	113.52	Non-dominated (frontier)	-

Note: Status is defined per texture: Baseline control = untreated comparator; Dominant (vs some) = cheaper and more effective than at least one alternative; Dominated = more expensive and less (or no more) effective  $\rightarrow$  excluded; Non-dominated (frontier) = option remaining on the efficient set after removing dominated options; Extended dominated = excluded because its incremental ICER ( $\Delta\text{Cost}/\Delta\text{Effect}$  between adjacent non-dominated options) exceeds that of a more effective neighbor (as observed in Sandy: NG-2).

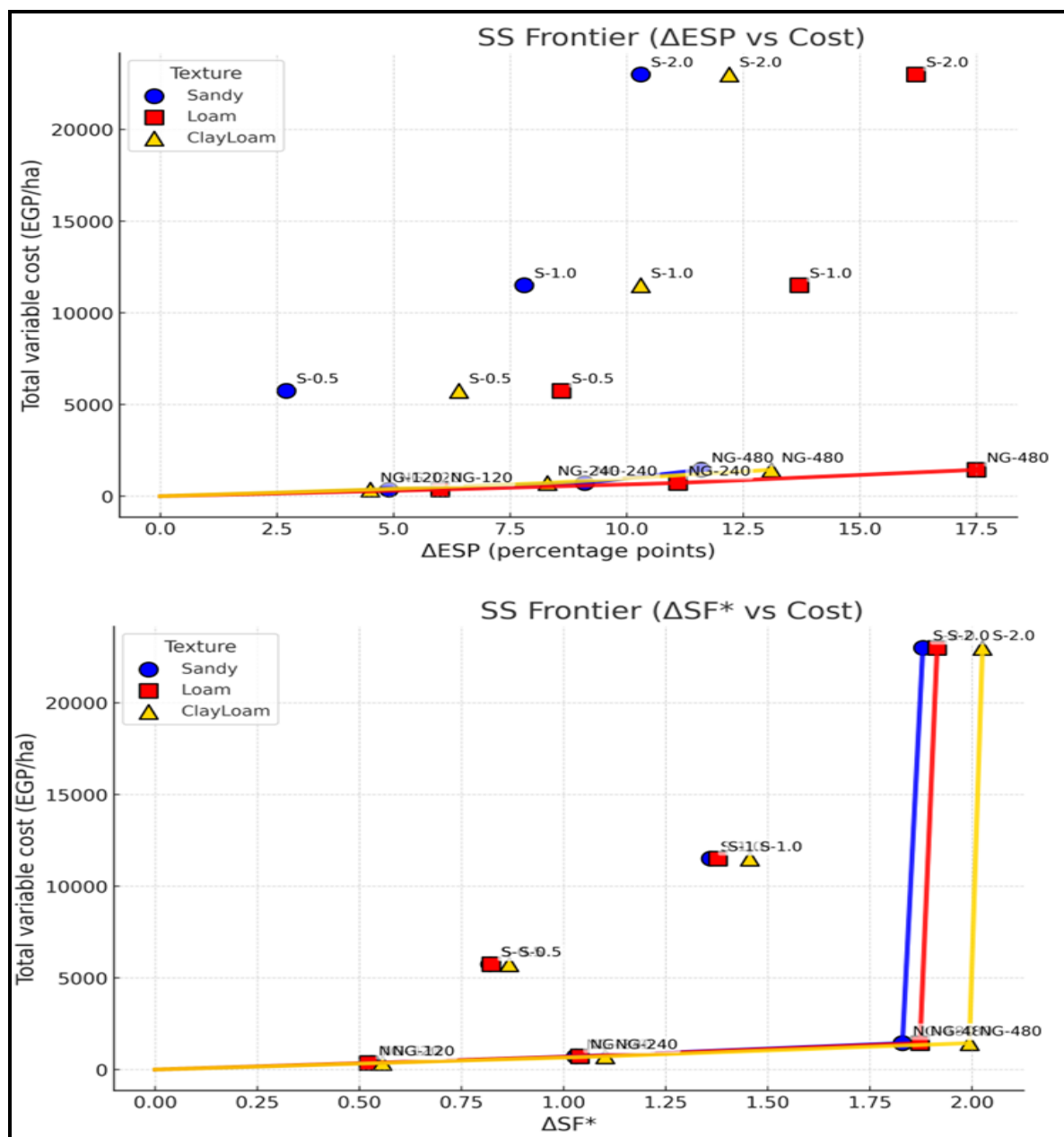


Fig. 12. Cost-effectiveness frontiers for saline-sodic (SS) soils at six months, showing  $\Delta\text{ESP}$  and  $\Delta\text{SF}^*$  relative to total variable cost ( $\text{EGP ha}^{-1}$ ), by soil texture

### Cost and Benefit Efficiency (six-month period; by texture)

The willingness-to-pay parameter ( $\lambda$ ) and the net-benefit form are defined as

Net Benefit =  $\lambda \times \Delta\text{ESP} - \text{Cost}$ , with  $\lambda$  informed by a threshold-slope (broken-line) yield model. The conservative base case is  $\lambda = 200$  EGP ha<sup>-1</sup> per ESP point, with sensitivity runs at  $\lambda = 500$  and  $\lambda = 1000$ .

**Saline-sodic (SS) soils.** By month 6, all NG rates yield positive net benefits across textures, whereas all S<sup>0</sup> rates remain negative at  $\lambda = 200$ , reflecting higher material/application costs and slower oxidation; ICER/frontier analysis classifies NG options on the efficient set, with S-1/S-2 strictly dominated and S-3 appearing on the frontier only at very steep ICERs (poor six-month value). Extended dominance is noted (e.g., NG-2 weakly dominated in sandy). Loam SS (an example). NG options remain non-dominated (frontier) with low cost per unit effect, while S-1/S-2 are dominated; S-3 can be non-dominated but at  $\approx 120$  EGP per 0.01 SF\*, orders of magnitude higher than NG.

#### Non-saline sodic (NSS) soils.

**Cost-benefit.** At  $\lambda = 200$ , NG achieves positive NB in sandy, loam, and clay-loam, while S<sup>0</sup> is negative or near break-even; at  $\lambda = 500$  and  $\lambda = 1000$ , NG's NB expands strongly whereas S<sup>0</sup> often remains unattractive within six months (e.g., sandy NSS, NG-1/-2/-3  $\rightarrow$  +1,340/+2,430/+3,510 at  $\lambda=500$ ; +3,040/+5,580/+8,460 at  $\lambda=1000$ ).

**Cost-effectiveness (SF\* ICER).** Across textures, NG-2 and NG-3 repeatedly sit on the frontier with low cost per 0.01 SF\* ( $\approx 6-8$  EGP), whereas S-1/S-2 are dominated; S-3 may be non-dominated in some cases but typically at  $\approx 120$  EGP per 0.01 SF\*.

**Implication.** Across both NSS and SS categories, the economically efficient six-month strategy is moderate NG ( $\approx 240-480$  kg ha<sup>-1</sup>) in all textures. Elemental sulfur becomes a candidate mainly for longer periods and/or higher  $\lambda$  scenarios (high-value, sensitive crops) where delayed acidification benefits can amortize higher upfront costs—consistent with the mechanistic and dominance findings.

### F. Recommendations by Soil Class (NSS vs SS) and Texture

#### 1) Non-saline sodic (NSS)

**Goal:** ESP < 15% (desodification) with concurrent structural gains ( $\uparrow K^+$ ,  $\uparrow AW$ ,  $\uparrow SF^*$ ).

**Amendment choice:** Prioritize nano-gypsum (NG) for rapid Ca<sup>2+</sup> supply and early structure recovery; consider elemental sulfur (S<sup>0</sup>) as a slower, pH-directed complement in high-CaCO<sub>3</sub>, fine textures.

• **Sandy NSS ( $\approx 8\%$  CaCO<sub>3</sub>):** Six-month data show strong ESP/SAR decline (e.g., ESP 17.8%  $\rightarrow$  7.9% at NG-480; S-2000: 8.6%; SAR 12.7  $\rightarrow$  6.7), with ECe remaining < 4. Start with NG-240; use **NG-480** if baseline ESP is high or if rapid structural gains are required.

• **Loam NSS ( $\approx 15\%$  CaCO<sub>3</sub>):** ESP reached single digits at higher NG/S (e.g., NG-480 8.8%; S-2000 9.6%). **Recommend NG-240** as the default; escalate to **NG-480** when initial ESP  $\geq 20\%$  or when early SF\* recovery is critical.

• **Clay-loam NSS ( $\approx 28\%$  CaCO<sub>3</sub>):** Despite strong buffering, ESP met the < 15% target by six months (e.g., NG-480 10.6%; S-2000 11.5%). **Use NG-480 first**; where alkalinity remains high, **add S<sup>0</sup>** in a staged program.

**Monitoring & action:** If ESP plateaus > 15% at three months, **repeat NG-240** and maintain a leaching fraction; **add S<sup>0</sup>** in carbonate-rich clay-loam where pH dampens Ca<sup>2+</sup> activity.

#### 2) Saline-sodic (SS)

**Goals:** ESP < 15% and ECe < 4 dS m<sup>-1</sup> (may require > 6 months in finer textures). Pair amendment with drainage + leaching to push salinity below threshold.

• **Sandy SS:** Fastest chemical/physical response. Six-month cost-benefit shows positive net benefit for all NG rates at  $\lambda = 200$  and S-1/S-2 dominated; NG-240 gives the best value, with NG-480 for maximum  $\Delta\text{ESP}/\Delta\text{SF}^*$ .

**Physical dynamics:** early K<sup>+</sup> jump, then consolidation of AW and SF\* by month 6—consistent with effective Ca-Na exchange and salt leaching.

• **Loam SS:** Reaches ESP  $\approx 14-15\%$  at higher NG/S but salinity can remain > 4 dS m<sup>-1</sup> at six months. Favor NG-240/NG-480 and extend leaching; S-1/S-2 remain dominated, S-3 is on the frontier but costly (high ICER).

• **Clay-loam SS:** At six months, ESP often > 15% and ECe high despite treatment; choose NG-480 as the staging dose on the frontier, plan follow-on NG-240 or NG + S<sup>0</sup> with reinforced leaching/drainage until thresholds are met. (*Frontier: NG-1/-2/-3 non-dominated; S-1/S-2 dominated; S-3 on frontier but high ICER.*)

**Mechanistic note:** NG's six-month advantage is expected (immediate Ca<sup>2+</sup>  $\rightarrow$  flocculation, pore reconnection); S<sup>0</sup> contributes more gradually via acid dissolution of CaCO<sub>3</sub>—useful in carbonate-rich, fine textures over a longer time.

### Site Conditions and Operational Context (Siwa)

- **Irrigation-water quality (inlet):** Mean EC<sub>w</sub> ≈ 3.75 dS m<sup>-1</sup> (range 2.27–5.46) and SAR ≈ 6.26 (range 4.33–8.08) across the study; one common supply for all plots. This EC–SAR combination implies limited dispersion risk during application (electrolyte aids flocculation) but can drive **ESP rebound** over time without Ca<sup>2+</sup> inputs and leaching.
- **Soil & hydro-geomorphic setting:** Highly calcareous soils under shallow groundwater with waterlogging risk; reclamation therefore hinges on amendment + controlled leaching under functional drainage.
- **Plot layout & hydraulics:** Plots oriented across local slope, bounded by ~30 cm bunds with buffer strips and alleys to limit lateral flow and preserve drainage pathways—conditions that frame how leaching is applied and interpreted.

### Operational protocol (Siwa field procedures)

1. **Apply amendment → leach.** Broadcast/incorporate to 0–20 cm, then apply ~300 mm leaching irrigation in multiple passes to dissolve CaSO<sub>4</sub> and flush Na<sub>2</sub>SO<sub>4</sub>.
2. **Drainage first.** Maintain ditches/drains to lower the shallow water table and prevent ponding; retain bunds/buffers to minimize lateral flow (slope-normal plots and protected boundaries).
3. **Monitor at 1, 3, and 6 months.** Track ESP, SAR, EC<sub>e</sub>, K<sub>s</sub>, AW, SF\* using the standardized sampling/core methods; if at 6 months ESP ≥ 15 % or (in SS) EC<sub>e</sub> ≥ 4 dS m<sup>-1</sup>, repeat NG (often 240 kg ha<sup>-1</sup>) and/or add S<sup>0</sup>, accompanied by additional leaching.

**Water-quality management.** With EC<sub>w</sub> ≈ 3–4 dS m<sup>-1</sup> and SAR ≈ 5–7, infiltration is generally stable during application, but continued use without Ca<sup>2+</sup> addition and planned leaching risks rising ESP—hence pairing irrigation with NG/S<sup>0</sup> and maintaining a leaching fraction until EC<sub>e</sub> trends downward.

### CONCLUSION

This study highlights the performance and cost-effectiveness of applying nano gypsum and elemental sulfur amendments calcareous salt-affected soils in Siwa oasis. Nano-gypsum proved to be a rapid and cost-effective amendment in reclaiming the studied soils. However, elemental sulfur showed slower, long-term benefits mainly in carbonate-rich, fine-textured soils. NG (240–480 kg ha<sup>-1</sup>) reduced ESP < 15% in non-saline sodic soils within six months. Saline–sodic sandy and loam soils improved, but clay-loam required extended leaching. NG consistently outperformed S<sup>0</sup> in

cost-effectiveness. Practical recommendation: prioritize NG for rapid desodification, with S<sup>0</sup> as a supplementary option for sustained acidification.

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## الملخص العربي

### كفاءة تطبيق النانو-جبس والكبريت العنصري في استصلاح الأراضي الملحية القلوية في واحة سيوة، مصر

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الوصول إلى  $ESP < 15\%$  خلال ستة أشهر، مع انخفاضات في SAR/ECe وتحسن في التوصيل الهيدروليكي (Ks) ومعامل البناء، مما يؤكد حدوث إزالة فعالة للصودية. في أراضي SS، خفّضت المعاملات الـ ESP و SAR و ECe، لكن التقدم كان محدودًا بحسب القوام؛ إذ وصلت الأراضي الرملية والطينية إلى  $14-15\% \approx ESP$ ، بينما بقيت الأراضي الطينية الطينية فوق الحدود ( $>15\%$ ) بعد ستة أشهر، مع بقاء ECe أعلى من 4 ديسيسمنز/م، مما يشير إلى الحاجة لمزيد من الغسيل. أظهر التحليل الاقتصادي أن معدلات NG بين 240-480 كجم/هكتار كانت باستمرار على جبهة الكفاءة الاقتصادية لكل من  $\Delta ESP$  و  $\Delta SF^*$ . في المقابل، قدمت معدلات  $S^0$  (0.5-1.0 طن/هكتار) حلولًا مسيطر عليها (dominated)، في حين أن معدل 2.0 طن/هكتار بقي أحيانًا غير مسيطر عليه لكن مع نسب تكلفة فاعلية حديثة مرتفعة (ICERs)، ما يجعله أقل جاذبية. نوصي بإعطاء الأولوية لاستخدام NG بمعدلات 240-480 كجم/هكتار لتحقيق خفض ESP واستعادة البناء خلال ستة أشهر. أما إضافة  $S^0 \leq 2$  طن/هكتار فيُنصح بها فقط في الأراضي الطينية الطينية الغنية بالكربونات للحفاظ على إذابة الكربونات بالحامض. ويجب اقتران هذه التطبيقات بالغسيل المباشر، وصيانة الصرف، ودورات غسيل ممتدة في أراضي SS حتى الوصول إلى الهدف  $ECe < 4$  ديسيسمنز/م. الكلمات الدالة: واحة سيوة، أراضى ملحية صودية، أراضى غير ملحية صودية، نانو جبس، كبريت عنصري.

تعد الأراضي الكلسية المتأثرة بالأملاح في واحة سيوة بمصر، والتي تتميز بارتفاع مستوى المياه الجوفية واعتمادها على الري الهامشي، بيئة محفوفة بمخاطر مزدوجة من الصودية والملوحة، مما يحد بشكل كبير من نمو المحاصيل. استهدفت هذه الدراسة تحديد معدلات الإضافة المثلى والفعالة من حيث التكلفة لكل من النانو-جبس (NG) والكبريت العنصري ( $S^0$ ) لتحقيق أهداف الاستصلاح خلال ستة أشهر. أجريت تقييم التكلفة لكل وحدة انخفاض في نسبة الصوديوم المتبادل (ESP) ولكل زيادة بمقدار 0.01 في معامل البناء ( $SF^*$ )، مع الأخذ في الاعتبار اختلافات الاستجابة للمعدل والمدة. تمت مقارنة معدلات NG (120، 240، 480 كجم/هكتار) و  $S^0$  (0.5، 1.0، 2.0 طن/هكتار) في أراضى صودية غير ملحية (NSS) وأراضى صودية - ملحية (SS) عبر قوامات رملية وطينية وطينية طينية. تمت خلط المحسنات حتى عمق 0-20 سم، تلاها غسيل مباشر. أُخذت عينات التربة بعد 1 و 3 و 6 أشهر لقياس الخصائص الكيميائية (pH)، التوصيل الكهربائي (Ece)، (ESP)، نسبة امتصاص الصوديوم (SAR) والفيزيائية (التوصيل الهيدروليكي (Ks)، الاحتفاظ بالرطوبة، الماء المتاح (AW)). كما تم اشتقاق معامل البناء (SF) وصيغته المعيارية ( $SF^*$ ). أُجري التحليل الإحصائي باستخدام تصميم القطاعات الكاملة العشوائية بثلاثة عوامل (القوام × المعاملة × الزمن) مع القياسات المكررة، إضافة إلى تحليل التباين ANOVA واختبار أقل فرق معنوي (LSD) عند مستوى معنوية 0.05. في أراضي NSS، نجحت جميع القوامات في